ADVANCED MATHEMATICS

Bachelor Degree in Aerospace Engineering, Universidad Carlos III de Madrid

November 27, 2019

Total duration: 1 hour

PROBLEM 1. (4 points)

Let

$$f(x) = \begin{cases} \dfrac{5x}{2} & \text{if} \quad 0 < x < \dfrac{2}{3}, \\ 3 - 2x & \text{if} \quad \dfrac{2}{3} < x < 1. \end{cases}$$

1.1 Find the cosine Fourier series for the function $f(x)$ on $0 < x < 1$.

1.2 Sketch the graphs of $f(x)$ and its cosine Fourier series on the whole real line, specifying convergence of the latter at relevant points.

PROBLEM 2. (6 points)

Consider the following Laplace problem on a ring, described using polar coordinates r and θ:

$$\begin{cases} \nabla^2 u = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2} = 0, & 1 < r < 2, \ -\pi < \theta < \pi, \\ u(1,\theta) = 0, & r = 1, \quad -\pi < \theta < \pi, \\ u(2,\theta) = \sin 3\theta, & r = 2, \quad -\pi < \theta < \pi. \end{cases}$$

2.1 Justify why the *zero-eigenvalue* eigenfunction of the equation associated with θ (as obtained by separation of variables) does not play any role in this particular problem.

2.2 Solve the problem, using part 2.1 to simplify the discussion.
