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# Midterm-3-Group-46.pdf

*Soluciones midterm 3 2018*

**2° Ampliación de Matemáticas**

**Grado en Ingeniería Aeroespacial**

**Escuela Politécnica Superior**
**UC3M - Universidad Carlos III de Madrid**

Como aún estás en la portada, es momento de redes sociales. Cotilléanos y luego a estudiar.

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# MIDTERM 3 2018 46

①

$$\begin{cases} \nabla(u) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, & 0<r<a, \quad 0<\theta<\frac{\pi}{3} \\ u(r,0) = 0, & 0<r<a \\ u(r,\frac{\pi}{3}) = 0, & 0<r<a \\ u(a,\theta) = \sin(3\theta) + 2\sin(9\theta) \end{cases}$$

$u(r,\theta) = \rho(r)\varphi(\theta)$

$\frac{1}{r}\frac{\partial}{\partial r}\left(r\varphi\frac{\partial \rho}{\partial r}\right) + \frac{\rho}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2} = 0$

$\frac{1}{r\rho}\frac{\partial}{\partial r}\left(r\frac{\partial \rho}{\partial r}\right) + \frac{1}{r^2\varphi}\frac{\partial^2 \varphi}{\partial \theta^2} = 0$

$\frac{r}{\rho}\frac{\partial}{\partial r}\left(r\frac{\partial \rho}{\partial r}\right) + \frac{1}{\varphi}\frac{\partial^2 \varphi}{\partial \theta^2} = 0 \rightarrow \frac{\varphi''}{\varphi} = -\frac{r}{\rho}\frac{\partial}{\partial r}\left(r\frac{\partial \rho}{\partial r}\right) = -\lambda$

$\begin{cases} \varphi'' = -\lambda\varphi \\ \varphi(0) = \varphi(\frac{\pi}{3}) = 0 \end{cases}$ → Dirichlet BC: $\lambda_n = \frac{n^2\pi^2}{(\pi/3)^2} = 9n^2$, $n = 1,2,3\ldots$

$\varphi_n(\theta) = \sin(3n\theta)$

$\frac{r}{\rho}\frac{\partial}{\partial r}\left(r\frac{\partial \rho}{\partial r}\right) = 9n^2$

$r\left[\frac{\partial \rho}{\partial r} + r\frac{\partial^2 \rho}{\partial^2 r}\right] = 9n^2\rho$

$r^2\rho'' + r\rho' - 9n^2\rho = 0$ → Euler's eq. → $\rho(r) = r^\alpha$

$r^2\alpha(\alpha-1)r^{\alpha-2} + r\alpha r^{\alpha-1} - 9n^2 r^\alpha = 0$

$r^\alpha(\alpha^2 - \cancel{\alpha} + \cancel{\alpha} - 9n^2) = 0 \rightarrow \alpha = \pm 3n$

$\rho(r) = A r^{3n} + \frac{B}{r^{3n}}$ → $B = 0$ to avoid singularity at origin

Generalized linear superposition principle:

$$\boxed{u(r,\theta) = \sum_{n=1}^{\infty} \rho_n(r)\varphi_n(\theta) = \sum_{n=1}^{\infty} A_n r^{3n}\sin(3n\theta)} \rightarrow \boxed{\left(\frac{r}{a}\right)^3\sin(3\theta) + 2\left(\frac{r}{a}\right)^9\sin(9\theta)}$$

($n=1$, $n=3$)

$u(a,\theta) = \sin(3\theta) + 2\sin(9\theta) = \sum_{n=1}^{\infty} A_n a^{3n}\sin(3n\theta)$

- $n=1 \rightarrow A_1 a^3 = 1 \rightarrow A_1 = \frac{1}{a^3} = a^{-3}$
- $n=3 \rightarrow A_3 a^1 = 2 \rightarrow A_3 = 2a^{-9}$
- $n \neq 1,3 \rightarrow A_n a^{3n} = 0 \rightarrow A_n = 0$

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Si esta página te inquieta, te atormenta o te perturba es porque he torcido el marco.
Si llegamos a 14k seguidores en Twitter vuelvo a ponerla bien.

(2)

a) $\begin{cases} x^2\varphi'' + x\varphi' + \lambda\varphi = 0 \\ \varphi(1) = 0, \ \varphi(e) = 0 \end{cases}$

Prove it is not S-L form and transform it into it.

S-L: $\frac{d}{dx}\left(p(x)\frac{d\varphi}{dx}\right) + q(x)\varphi + \sigma(x)\lambda\varphi = 0$

$p\varphi'' + p'\varphi' + q\varphi + \sigma\lambda\varphi = 0$

$x^2\varphi'' + x\varphi' + \lambda\varphi = 0$

$p(x) = x^2 \rightarrow p'(x) = 2x \neq x \rightarrow$ not S-L type

Transform it using an integrating factor:

$\mu x^2\varphi'' + \mu x\varphi' + \lambda\mu\varphi = 0 \quad ) \ 1/x^2$

$\mu\varphi'' + \frac{\mu}{x}\varphi' + \frac{\lambda}{x^2}\mu\varphi = 0$

$\frac{\mu}{x} = \mu' \rightarrow \frac{\mu'}{\mu} = \frac{1}{x} \rightarrow \ln\mu = \ln x$

$\mu(x) = x$

$\frac{d}{dx}(\mu\varphi') = \mu\varphi'' + \mu'\varphi'$

$\boxed{x\varphi'' + \varphi' + \frac{\lambda}{x}\varphi = 0}$ where $p(x) = x$; $q(x) = 0$; $\sigma(x) = \frac{1}{x}$

b) Using the Rayleigh quotient, prove that all the eigenvalues are positive

$\lambda = \frac{-p\varphi\frac{d\varphi}{dx}\Big|_1^e + \int_1^e \left[p\left(\frac{d\varphi}{dx}\right)^2 - q\varphi^2\right]dx}{\int_1^e \varphi^2\sigma\,dx} = \frac{\int_1^e x\left(\frac{d\varphi}{dx}\right)^2}{\int_1^e \frac{\varphi^2}{x}\,dx} \geq 0$ ✓

$(\varphi(e) - \varphi(1)) = 0 - 0 = 0$

(*) The Rayleigh quotient directly proves that $\lambda \geq 0$ if

a) $-p\phi\frac{d\phi}{dx}\Big|_a^b \geq 0$ and b) $q \leq 0$ (S.G.H)

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