ADVANCED MATHEMATICS

Bachelor Degree in Aerospace Engineering, Universidad Carlos III de Madrid

Group 46, course 2018-2019

November 30, 2018

Total duration: 1 hour

PROBLEM 1. (7 points)

Solve the following Laplace equation over a $60°$ wedge,

$$\begin{cases} \Delta(u) = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial u}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2} = 0, & 0 < r < a, \quad 0 < \theta < \pi/3, \\ u(r,0) = 0, & 0 < r < a, \\ u(r,\pi/3) = 0, & 0 < r < a, \\ u(a,\theta) = \sin(3\theta) + 2\sin(9\theta). \end{cases}$$

PROBLEM 2. (3 points)

a) Prove that the following equation is not of Sturm-Liouville type and transform it into this type

$$\begin{cases} x^2\varphi'' + x\varphi' + \lambda\varphi = 0, \\ \varphi(1) = 0, \quad \varphi(e) = 0. \end{cases}$$

b) Using the Rayleigh quotient, prove that all the eigenvalues are positive.
