ADVANCED MATHEMATICS

Bachelor Degree in Aerospace Engineering, Universidad Carlos III de Madrid

October 23, 2019

Total duration: 1 hour

PROBLEM 1. (4 points)

Given $\mathbf{x}(t) = (x(t),\ y(t))^T$, consider the system of equations

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} -3 & 1 \\ -1 & -1 \end{pmatrix} \mathbf{x}.$$

1.1 Find the eigenvalues and eigenvectors of the coefficient matrix occurring in the system of equations and sketch the phase portrait.

1.2 Find the matrix for the change of basis that puts the system into canonical form and write down the system in the new coordinates.

1.3 Solve the system in the coordinates mentioned in 1.2.

PROBLEM 2. (6 points)

Consider the following ecological model of two interacting species whose populations are x and y:

$$\begin{aligned} \frac{dx}{dt} &= x(4 - 2x + \alpha y), \\ \frac{dy}{dt} &= y(9 - 3y). \end{aligned}$$

2.1 Describe the interaction between the species for $\alpha < 0$, $\alpha = 0$ and $\alpha > 0$.

For items 2.2 to 2.4, we only consider the specific case in which $\boldsymbol{\alpha = 1}$.

2.2 Find the stationary points and determine their type.

2.3 Sketch the phase portrait.

2.4 Based on the phase portrait and your answer to 2.1, discuss the evolution of the two populations.
