ADVANCED MATHEMATICS

Bachelor Degree in Aerospace Engineering, Universidad Carlos III de Madrid

October 2, 2019

Total duration: 1 hour

PROBLEM 1. (5 points)

Consider Newton's law of cooling,

$$\frac{du}{dt} = -k[u - T(t)],$$

where $u(t)$ is the internal temperature of a body, and $T(t)$ is the ambient temperature, which grows linearly with time, $T(t) = T_0 + Ct$. All the parameters are positive, $k, T_0, C > 0$.

1.1 Are there any time-independent solutions?

1.2 Find the solution of the initial value problem in which $u(0) = u_0$, simplifying the expression as much as possible.

1.3 Discuss the evolution of $u(t)$ for $t \to \infty$, and its dependence on the initial value u_0. What is the effect of increasing the value of k?

PROBLEM 2. (5 points)

Consider the following non-homogeneous second-order differential equation with non-constant coefficients:

$$ty'' - (1 + t)y' + y = t^2\, e^{2t}.$$

2.1 Verify that the functions $y_1(t) = 1 + t$ and $y_2(t) = e^t$ are solutions of the corresponding homogeneous equation. Are they linearly independent? Why?

2.2 Find a particular solution, $y_p(t)$, of the full non-homogeneous equation.
