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Exámenes resueltos 2017/2018

2° Ampliación de Matemáticas

Grado en Ingeniería Aeroespacial

Escuela Politécnica Superior
Universidad Carlos III de Madrid



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ADVANCED MATHEMATICS

Bachelor Degree in Aerospace Engineering, Universidad Carlos III de Madrid

Groups 45, 46, and 47

18 June, 2018

Total duration 4 h.

PROBLEM 1. (2 points)

Suppose a particle of mass m is falling through a viscous fluid. There are two forces on the particle, gravity (with constant acceleration g) and fluid resistance. Assume that the latter is $-\gamma v^2$, where v is the particle velocity and $\gamma > 0$ is a constant. Consider v to be *positive* for a particle falling *downward*.

1.1 Write down a differential equation for $v(t)$ starting out from Newton's equation $m\,a(t) = F(t)$, where $a(t)$ is the particle acceleration and $F(t)$ is the net force acting on it.

1.2 Find the solution for $v(t)$ when $v(t=0) = V$.

1.3 Show that there is a so-called *terminal velocity*, v_T, that is approached at long times, and find its value. Interpret this fact physically.

PROBLEM 2. (2 points)

Find the solution for the initial value problem $\ddot{y} + 4\dot{y} + 4y = \sin t$, with $y(0) = \dot{y}(0) = 0$.

PROBLEM 3. (2 points)

Given the non-linear system $\begin{cases} \dot{x} &= x - x^2 - xy, \\ \dot{y} &= \dfrac{y}{2} - \dfrac{y^2}{4} - \dfrac{3xy}{4}, \end{cases}$

3.1 Find all the critical points (equilibrium solutions) and classify them.

3.2 Sketch the phase portrait.

PROBLEM 4. (2 points)

Solve the following Laplace problem on a circular sector (expressed in polar coordinates):

$$\begin{cases} \nabla^2 u = 0, & 0 < r < R,\ 0 < \theta < \pi/3, \\ u(r,0) = u(r,\pi/3) = 0, & 0 < r < R, \\ u(R,\theta) = f(\theta), \end{cases}$$

where $R > 0$ is constant and $f(\theta)$ is an arbitrary function.

[NOTE: Recall that, in polar coordinates, $\nabla^2 u = \dfrac{\partial^2 u}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial \theta^2}$.]

PROBLEM 5. (2 points)

Consider a heat-diffusion problem with convection: $\begin{cases} u_t = u_{xx} - u_x + xe^{-t}, & 0 < x < 1, \quad t > 0, \\ u(0,t) = 0 = u(1,t), & t > 0, \\ u(x,0) = 1 - x, & 0 < x < 1. \end{cases}$

5.1 Find the eigenvalues and eigenfunctions of the associated Sturm-Liouville problem.

5.2 Determine the solution of the full non-homogeneous problem providing as much detail as possible.

5.3 Discuss qualitatively the physical behavior of the solution.

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① 

a) $m\ddot{x} = mg - \gamma \dot{x}^2$ $\quad \dot{x} = v$

$m\dot{v} = mg - \gamma v^2$

$\gamma v^2 = mg - m\dot{v}$

$v^2 = \frac{m}{\gamma} g - \frac{m}{\gamma} \dot{v}$ $\rightarrow$ $\boxed{v(t) = \pm \sqrt{\frac{m}{\gamma}\left(g - \frac{dv}{dt}\right)}}$

b)

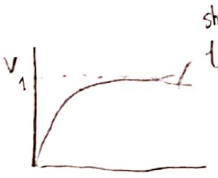
$$\begin{cases} v^2 = \frac{m}{\gamma} g - \frac{m}{\gamma}\frac{dv}{dt} \rightarrow \frac{dv}{dt} + \frac{\gamma}{m} v^2 = g \rightarrow \int_V^{v(t)} \frac{dv}{g - \frac{\gamma}{m} v^2} = \int_0^t dt \\ v(t=0) = V \end{cases}$$

$$\int \frac{dv}{g - \frac{\gamma}{m} v^2} = \int \frac{dv \cdot m/\gamma}{\frac{m}{\gamma}\left(g - \frac{\gamma}{m} v^2\right)} = \frac{m}{\gamma} \int \frac{dv}{\frac{mg}{\gamma} - v^2} = \frac{m}{\gamma} \int \frac{dv}{\left(\sqrt{\frac{mg}{\gamma}}\right)^2 - v^2} = \frac{m}{\gamma} \frac{1}{\sqrt{\frac{mg}{\gamma}}} \operatorname{arctanh}\left(\frac{v}{\sqrt{mg/\gamma}}\right) =$$

$$= \sqrt{\frac{m}{\gamma g}} \operatorname{arctanh}\left(\sqrt{\frac{\gamma}{mg}} v\right) - \sqrt{\frac{m}{\gamma g}} \operatorname{arctanh}\left(\sqrt{\frac{\gamma}{mg}} V\right) = t$$

$$\operatorname{arctanh}\left(\sqrt{\frac{\gamma}{mg}} v\right) = \sqrt{\frac{\gamma g}{m}} t + \operatorname{arctanh}\left(\sqrt{\frac{\gamma}{mg}} V\right)$$

$$\boxed{v(t) = \sqrt{\frac{mg}{\gamma}} \tanh\left(\sqrt{\frac{\gamma g}{m}} t + \operatorname{arctanh}\left(\sqrt{\frac{\gamma}{mg}} V\right)\right)}$$

c) 

$$\lim_{t \to \infty} \sqrt{\frac{mg}{\gamma}} \underbrace{\tanh\left(\sqrt{\frac{\gamma g}{m}} t + \operatorname{arctanh}\left(\sqrt{\frac{\gamma}{mg}} V\right)\right)}_{\to 1} = \boxed{\sqrt{\frac{mg}{\gamma}} = v_t}$$

When terminal velocity is reached $\rightarrow m\ddot{x} = mg - \gamma \frac{mg}{\gamma} = 0 \rightarrow$

$\rightarrow$ the acceleration is zero (Fg and Fv are at equilibrium) $\rightarrow$ MRU

② $\begin{cases} \ddot{y} + 4\dot{y} + 4y = \sin t \\ y(0) = \dot{y}(0) = 0 \end{cases}$

- Homogeneous

$k^2 + 4k + 4 = 0 \rightarrow k = \frac{-4 \pm \sqrt{16 - 16}}{2} = -2$ double

$y_H(t) = Ae^{-2t} + Bte^{-2t}$

- Particular:

$y_p(t) = C\cos t + D\sin t$

$\dot{y}_p = -C\sin t + D\cos t$

$\ddot{y}_p = -C\cos t - D\sin t$

$\rightarrow$ $-C\cos t - D\sin t - 4C\sin t + 4D\cos t + 4C\cos t + 4D\sin t = \sin t$

$$\begin{cases} -C + 4D + 4C = 0 \rightarrow 3C + 4D = 0 \rightarrow \left(D = -\frac{3}{4}C = 3/25\right) \\ -D - 4C + 4D = 1 \end{cases}$$

$3D - 4C = 1$

$-3 \cdot \frac{3}{4} C - 4C = 1 \rightarrow -\frac{9}{4} C - 4C = 1 \rightarrow -\frac{25}{4} C = 1 \rightarrow \left(C = -4/25\right)$

$y_p(t) = -\frac{4}{25}\cos t + \frac{3}{25}\sin t$

line

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$y(t) = y_h + y_p = Ae^{-2t} + Bte^{-2t} + \frac{1}{25}(-4\cos t + 3\sin t)$

$\dot{y}(t) = -2Ae^{-2t} - 2Bte^{-2t} + Be^{-2t} + \frac{1}{25}(4\sin t + 3\cos t)$

$y(0) = A - \frac{4}{25} = 0 \rightarrow A = \frac{4}{25}$

$\dot{y}(0) = -2A + B + \frac{3}{25} = 0 \rightarrow \left(B = 2A - \frac{3}{25} = \frac{1}{5}\right)$

$$y(t) = \frac{4}{25}e^{-2t} + \frac{1}{5}te^{-2t} - \frac{4}{25}\cos t + \frac{3}{25}\sin t$$

③ $\begin{cases} \dot{x} = x - x^2 - xy = x(1-x-y) \\ \dot{y} = \frac{y}{2} - \frac{y^2}{4} - \frac{3xy}{4} = y\left(\frac{1}{2} - \frac{y}{4} - \frac{3}{4}x\right) \end{cases}$

a) · (0,0)

· $x=0$: $\frac{1}{2} - \frac{y}{4} - \frac{3}{4}\cancel{x}^{0} = 0 \rightarrow \frac{y}{4} = \frac{1}{2} \rightarrow y = 2 \rightarrow (0,2)$

· $y=0$: $1 - x - \cancel{y}^{0} = 0 \rightarrow x = 1 \rightarrow (1,0)$

· $\begin{cases} 1-x-y=0 \rightarrow y = 1-x = 1-\frac{1}{2} = \frac{1}{2} \\ \frac{1}{2} - \frac{y}{4} - \frac{3}{4}x = 0 \end{cases} \rightarrow \left(\frac{1}{2}, \frac{1}{2}\right)$

$\frac{1}{2} - \frac{1-x}{4} - \frac{3}{4}x = 0 \rightarrow \frac{1}{2} = \frac{1}{4} + \frac{x}{4} - \frac{3}{4}x = 0 \rightarrow \frac{1}{2}x = \frac{1}{4} \rightarrow x = \frac{1}{2}$

$$DF(x,y) = \begin{pmatrix} 1-2x-y & -x \\ -\frac{3}{4}y & \frac{1}{2} - \frac{y}{2} - \frac{3x}{4} \end{pmatrix}$$

· $DF(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ $\lambda_1 = 1$ $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\lambda_2 = \frac{1}{2}$ $\vec{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ → unstable node

$|\lambda_2| < |\lambda_1|$ → tangent to $\vec{v}_2$

· $DF(0,2) = \begin{pmatrix} -1 & 0 \\ -3/2 & -1/2 \end{pmatrix}$ $\lambda_1 = -1 \rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ $\lambda_2 = -\frac{1}{2} \rightarrow \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ → stable node

→ $|\lambda_2| < |\lambda_1|$ → tangent to $\vec{v}_2$

$\lambda_1 = -1$

$\begin{pmatrix} 0 & 0 \\ -3/2 & 1/2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow -\frac{3}{2}a + \frac{1}{2}b = 0 \rightarrow \frac{b}{2} = \frac{3}{2}a \rightarrow b = 3a \rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

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$\cdot DF(1,0) = \begin{pmatrix} -1 & -1 \\ 0 & -1/4 \end{pmatrix}$ $\lambda_1 = -1 \to \vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\lambda_2 = -\frac{1}{4} \to \vec{v}_2 = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

stable node

$|\lambda_2| < |\lambda_1|$ → tangent to $\vec{v}_2$

$\lambda_2 = -1/4$

$\begin{pmatrix} -3/4 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow -\frac{3}{4}a - b \Rightarrow b = -\frac{3}{4}a$ $\vec{v}_2 = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$\cdot DF\left(\frac{1}{2}, \frac{1}{2}\right) = \begin{pmatrix} -1/2 & -1/2 \\ -3/8 & -1/8 \end{pmatrix} = -\frac{1}{8}\begin{pmatrix} 4 & 4 \\ 3 & 1 \end{pmatrix}$

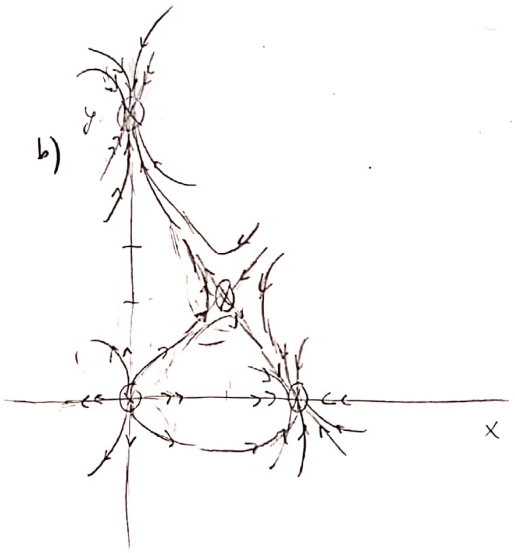
$\begin{vmatrix} 4-\lambda & 4 \\ 3 & 1-\lambda \end{vmatrix} = 4+\lambda^2-\lambda-4\lambda-12 = \lambda^2-5\lambda-8=0$

$\lambda_1 = \left(\frac{5+\sqrt{57}}{2}\right)\cdot\left(-\frac{1}{8}\right) < 0$

$\lambda_2 = \left(\frac{5-\sqrt{57}}{2}\right)\cdot\left(-\frac{1}{8}\right) > 0$

→ saddle point

b)



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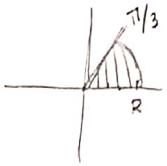
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(4)

$$\begin{cases} \nabla^2 u = 0 & 0<r<R,\ 0<\theta<\pi/3 \\ u(r,0) = u(r,\frac{\pi}{3}) = 0 & 0<R \\ u(R,\theta) = f(\theta) \end{cases}$$



$u(r,\theta) = \rho(r)\,\phi(\theta)$

$$\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}$$

$$\phi\rho'' + \frac{\phi}{r}\rho' + \frac{\rho}{r^2}\phi'' = 0$$

$$\frac{\rho''}{\rho} + \frac{1}{r}\frac{\rho'}{\rho} + \frac{1}{r^2}\frac{\phi''}{\phi} = 0$$

$$r^2\frac{\rho''}{\rho} + r\frac{\rho'}{\rho} = -\frac{\phi''}{\phi} = \lambda$$

$$\begin{cases} \phi'' = -\lambda\phi \\ \phi(0) = \phi(\frac{\pi}{3}) = 0 \rightarrow \text{Dirichlet BC} \end{cases}$$

$$\lambda_n = \frac{n^2\pi^2}{(\pi/3)^2} = 9n^2 \qquad n = 1,2\ldots$$

$$\phi_n(\theta) = \sin\left(\frac{n\pi}{\pi/3}\theta\right) = \sin(3n\theta)$$

$$r^2\frac{\rho''}{\rho} + r\frac{\rho'}{\rho} - \lambda = 0$$

$$r^2\rho'' + r\rho' - 9n^2\rho = 0 \rightarrow \text{Euler-Cauchy eq.} \quad r^\alpha$$

$$r^2\alpha(\alpha-1)r^{\alpha-2} + r\alpha r^{\alpha-1} - 9n^2 r^\alpha = 0$$

$$r^\alpha(\alpha^2 - \cancel{\alpha} + \alpha - 9n^2) = 0 \rightarrow \alpha = \pm 3n$$

$$\rho(r) = A_n r^{3n} + \cancel{\frac{B}{r^{3n}}} \quad 0 \text{ avoid singularity}$$

$$\boxed{u(r,\theta) = \sum_{n=1}^{\infty} A_n r^{3n}\sin(3n\theta)}$$

$$u(R,\theta) = f(\theta) = \sum_{n=1}^{\infty} A_n R^{3n}\sin(3n\theta)$$

$$\| \qquad \rightarrow \boxed{A_n = \frac{f_n}{R^{3n}} = \frac{\frac{2}{\pi/3}\int_0^{\pi/3} f(\theta)\sin(3n\theta)\,d\theta}{R^{3n}}}$$

$$\sum_{1}^{\infty} f_n \sin(3n\theta)$$

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(5)

$$\begin{cases} u_t = u_{xx} - u_x + xe^{-t} & , \ 0<x<1 \ , \ t>0 \\ u(0,t) = 0 = u(1,t) & , \ t>0 \\ u(x,0) = 1-x & , \ 0<x<1 \end{cases}$$

a) L:

b)

1) Homog.

$u_t = u_{xx} - u_x$

$u(x,t) = G(t)\phi(x)$

$G'\phi = G\phi'' - G\phi'$

$\frac{G'}{G} = \frac{\phi''}{\phi} - \frac{\phi'}{\phi} = -\lambda$

$\phi'' - \phi' + \lambda\phi = 0$

$\mu\phi'' - \mu\phi' + \lambda\mu\phi = 0$ $\quad \mu' = -\mu \rightarrow \frac{\mu'}{\mu} = -1 \rightarrow \ln\mu = -x \rightarrow \mu = e^{-x}$

$\frac{d}{dx}(\mu\phi') = \mu\phi'' + \mu'\phi'$

$\phi'' - \phi' + 1\phi = 0$

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$\phi'' - \phi' + \lambda\phi = 0$

$k^2 - k + \lambda = 0 \rightarrow k = \frac{1 \pm \sqrt{1-4\lambda}}{2}$

1.1) $1 - 4\lambda > 0 \rightarrow 1 > 4\lambda \rightarrow \lambda < \frac{1}{4}$ ~~$\lambda < \frac{1}{4}$~~ $\sqrt{}$ positive $k_\pm$

$\phi(x) = Ae^{k_+ x} + Be^{k_- x}$

$\phi(0) = 0 = A + B \rightarrow B = -A$

$\phi(1) = 0 = Ae^{k_+} + Be^{k_-} = A(e^{k_+} - e^{k_-}) = Ae^{1/2}\left(e^{\frac{\sqrt{1-4\lambda}}{2}} - e^{-\frac{\sqrt{1-4\lambda}}{2}}\right) =$

$= 2Ae^{1/2}\sinh\left(\frac{\sqrt{1-4\lambda}}{2}\right) \rightarrow A = 0 = B \rightarrow$ NO

$\neq 0$

1.2) $1 - 4\lambda = 0 \rightarrow$ ~~$\lambda = \frac{1}{4}$~~ $k = \frac{1}{2}$

$\phi(x) = Ae^{x/2} + Bxe^{x/2}$

$\phi(0) = A = 0$

$\phi(1) = \cancel{Ae^{1/2}}^{0} + Be^{1/2} = 0 \rightarrow B = 0$ } NO

1.3) $1 - 4\lambda < 0 \rightarrow 1 < 4\lambda \rightarrow \lambda > 1/4$ $\sqrt{}$ negative $k = \frac{1}{2} \pm \frac{i}{2}\sqrt{4\lambda - 1}$ ω

$\phi(x) = e^{x/2}\left(A\cos(\omega x) + B\sin(\omega x)\right)$

$\phi(0) = 0 = A$

$\phi(1) = e^{1/2} B\sin(\omega) \rightarrow \omega = n\pi \rightarrow \frac{\sqrt{4\lambda - 1}}{2} = n\pi \rightarrow 4\lambda - 1 = 4n^2\pi^2$

$4\lambda = 4n^2\pi^2 + 1$

$\lambda = n^2\pi^2 + \frac{1}{4}$ $\quad n = 1, 2, \ldots$

$\phi_n(x) = e^{x/2}\sin(n\pi x)$

2) ~~$\lambda = 0$~~

$\phi'' - \phi' = 0$

$k^2 - k = 0 \rightarrow k(k-1) = 0 \begin{cases} k = 0 \\ k = 1 \end{cases}$

$\phi(x) = A + Be^x$

$\phi(0) = 0 = A + B \rightarrow B = -A$

$\phi(1) = 0 = A + Be = A - Ae = A(1-e) \rightarrow A = B = 0 \rightarrow$ NO

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3) ~~1 < 0~~

$\phi'' - \phi' + \lambda\phi = 0$

$k^2 - k + \lambda = 0 \rightarrow k = \frac{1 \pm \sqrt{1-4\lambda}}{2}$ ← positive

λ negative

$\sqrt{1-4\lambda} > 0$ always

$\phi(x) = Ae^{k_+ x} + Be^{k_- x}$

$\phi(0) = 0 = A + B \rightarrow B = -A$

$\phi(1) = 0 = Ae^{k_+} + Be^{k_-} = A(e^{k_+} - e^{k_-}) = Ae^{1/2}\left(e^{\frac{\sqrt{1-4\lambda}}{2}} - e^{-\frac{\sqrt{1-4\lambda}}{2}}\right) =$

$= 2Ae^{-1/2}\sinh\left(\frac{\sqrt{1-4\lambda}}{2}\right) \rightarrow A = 0 = B \rightarrow$ NO

$\neq 0$

5)

1) Already done

2) $xe^{-t} = \sum_1^\infty F_n(t)\,\phi_n(x) \rightarrow F_n(t) = \frac{\langle \phi, f \rangle}{\langle \phi, \phi \rangle} = \frac{\int_0^1 xe^{-t}e^{x/2}\sin(n\pi x)\,e^{-x}\,dx}{\int_0^1 (e^{x/2})^2 \sin^2(n\pi x)\,e^{-x}\,dx} =$ ← $\sigma(x)$

$= e^{-t}\frac{\int_0^1 x e^{-x/2}\sin(n\pi x)}{\int_0^1 \sin^2(n\pi x)\,dx} = e^{-t} f_n$ ← independent of t

f_n ← $1/2$

3) $u(x,t) = \sum_1^\infty u_n(t)\,\phi_n(x)$

$\sum_1^\infty u_n'\phi = \sum_1^\infty u_n(\phi'' - \phi') + \sum_1^\infty e^{-t} f_n \phi_n$

$\phi'' - \phi' = -\lambda_n \phi_n$

$u_n' = -\lambda_n u_n + e^{-t} f_n$

$u_n' = -\left(n^2\pi^2 + \frac{1}{4}\right) u_n + e^{-t} f_n$

$u_n(0) = q_n$

$u(x,0) = 1 - x = \sum_1^\infty u_n(0)\,\phi_n(x)$

$\sum_1^\infty q_n \phi_n \quad u_n(0) = q_n = \frac{\int_0^1 (1-x)e^{-x/2}\sin(n\pi x)\,dx}{\int_0^1 \sin^2(n\pi x)\,dx}$

$1/2$

$u_n(t) = u_n(0)\,e^{-(n^2\pi^2 + \frac{1}{4})(t-0)} + \int_0^t e^{-(n^2\pi^2 + \frac{1}{4})(t-s)}\,e^{-s} f_n\,ds =$

$= q_n e^{-(n^2\pi^2 + \frac{1}{4})t} + f_n \int_0^t e^{-(n^2\pi^2 + \frac{1}{4})(t-s)}\,e^{-s}\,ds$

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$$u(x,t) = \sum_{n=1}^{\infty} u_n(t)\,\phi_n(x) \qquad \phi_n(x) = e^{x/2}\sin(n\pi x)$$

↓ ya resuelto

3)

$$u(x,t) = \sum_{n=1}^{\infty}\left[g_n e^{-(n^2\pi^2+\frac{1}{4})t} + f_n \int_0^t e^{-(n^2\pi^2+\frac{1}{4})(t-s)} e^{-s}\,ds \right] e^{x/2}\sin(n\pi x)$$

3) $$\int_0^t e^{-\lambda_n(t-s)} e^{-s}\,ds = \int_0^t e^{-\lambda_n t} e^{\lambda_n s} e^{-s}\,ds = e^{-\lambda_n t}\int_0^t e^{s(\lambda_n-1)}\,ds =$$

$$= e^{-\lambda_n t}\left[\frac{e^{s(\lambda_n-1)}}{(\lambda_n-1)}\right]_0^t = \frac{e^{-\lambda_n t}}{\lambda_n-1}\left[e^{t(\lambda_n-1)} - 1\right] = \frac{e^{-\lambda_n t}}{\lambda_n-1}\left[e^{\lambda_n t}e^{-t} - 1\right] =$$

$$= \frac{1}{\lambda_n-1}\left[e^{-t} - e^{-\lambda_n t}\right] \rightarrow u_n(t) = g_n e^{-\lambda_n t} + \frac{f_n}{\lambda_n-1}e^{-t} - \frac{f_n}{\lambda_n-1}e^{-\lambda_n t}$$

$$\lim_{t\to\infty} u(x,t) = 0$$

since $\lim_{t\to\infty} g_n e^{-\lambda_n t} = 0$ and $\lim_{t\to\infty} \frac{f_n}{1-\lambda_n}\left[e^{-t} - e^{-\lambda_n t}\right] = 0$

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t) \qquad u_n(t) = \left(g_n - \frac{f_n}{\lambda_n-1}\right)e^{-\lambda_n t} + \frac{f_n}{\lambda_n-1}e^{-t}$$

$$u_n(t) = g_n e^{-\lambda_n t} + \frac{f_n}{\lambda_{n-1}}\left(e^{-t} - e^{-\lambda_n t}\right) \rightarrow \text{Sale al Moviha} \checkmark$$

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other method.

b) $u_n' = -\lambda_n u_n + f_n e^{-t} \qquad u_n(0) = g_n$

• Homogeneous:

$u_n'' + \lambda_n u_n'' = 0 \rightarrow K + \lambda_n = 0 \rightarrow K = -\lambda_n$

$u_n''(t) = U_n e^{-\lambda_n t}$

• Particular

$u_n^p = B_n e^{-t} \rightarrow -B_n e^{-t} + \lambda_n B_n e^{-t} = e^{-t} f_n$

$u_n^{p\prime} = -B_n e^{-t} \qquad -B_n + \lambda_n B_n = f_n \rightarrow B_n = \frac{f_n}{\lambda_n - 1}$

$u_n(t) = U_n e^{-\lambda_n t} + \frac{f_n}{\lambda_n - 1} e^{-t}$

$u_n(0) = g_n = U_n + \frac{f_n}{\lambda_n - 1} \rightarrow U_n = g_n - \frac{f_n}{\lambda_n - 1}$

$$u_n(t) = \left(g_n - \frac{f_n}{\lambda_n - 1}\right) e^{-\lambda_n t} + \frac{f_n}{\lambda_n - 1} e^{-t}$$

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t)\, e^{x/2} \sin(n\pi x)$$

as $t \to \infty : u \to 0$

June

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