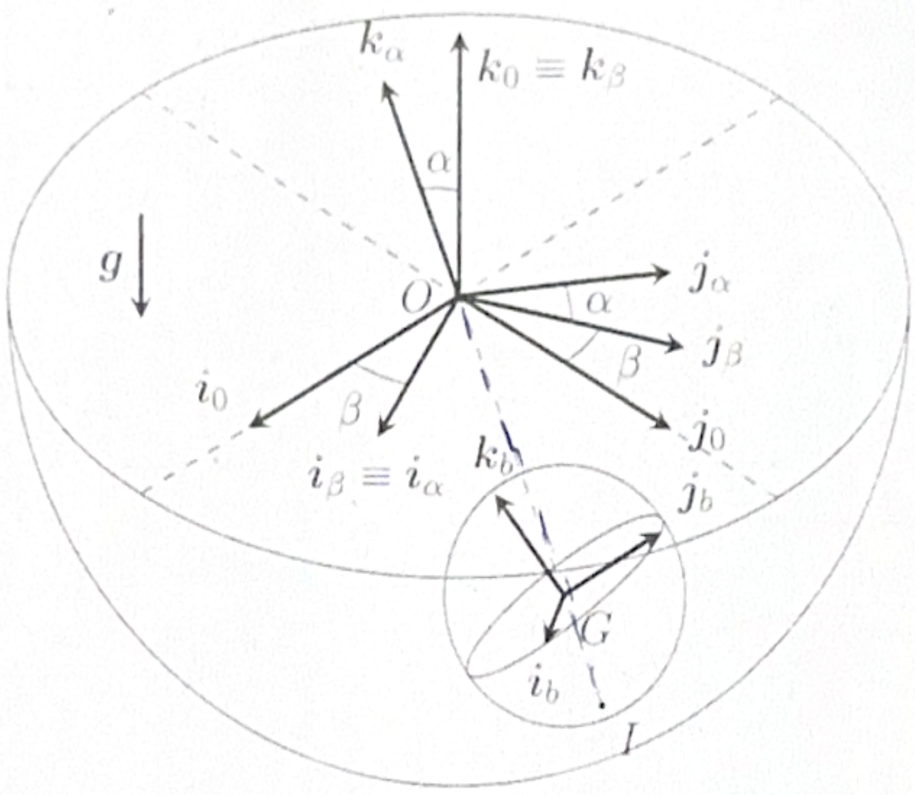
NAME: .....

NIA: ..... GROUP .....

A homogeneous, heavy, rigid ball b of mass m, radius R, and center G is in contact with the interior surface of a hollow hemisphere of radius $a > R$ and center O. We call I the instantaneous point of contact among the two. The moment of inertia of the ball about any axis that passes through G is $I_G^b = 2mR^2/5$. Assume the friction coefficient μ is infinite, so the ball rolls without slipping and, additionally, it cannot spin about an axis perpendicular to the instantaneous contact plane. The hollow hemisphere is fixed in an inertial reference frame S_0 with origin at O. To describe the motion of the ball, we define a body-fixed reference frame S_b with origin at G. The figure shows the system at a generic instant of time.



The position of G with respect to S_0 is given in terms of two angles α, β: the two angles define the auxiliary reference frames $S_\beta : \{O; \boldsymbol{i}_\beta, \boldsymbol{j}_\beta, \boldsymbol{k}_\beta\}$ and $S_\alpha : \{O; \boldsymbol{i}_\alpha, \boldsymbol{j}_\alpha, \boldsymbol{k}_\alpha\}$; the vector $\overrightarrow{OG}$ is parallel to the $\boldsymbol{k}_\alpha$ vector shown. The orientation of S_b with respect to S_α is given by a set of standard 3-1-3 Euler angles ψ, θ, and ϕ (not shown in the figure to avoid clutter).

Advice: draw a diagram of the $\{O; \boldsymbol{j}_\alpha, \boldsymbol{k}_\alpha\}$ plane (and separate diagrams for the simple rotations in the problem as needed). Work preferentially in the $\{\boldsymbol{i}_\alpha, \boldsymbol{j}_\alpha, \boldsymbol{k}_\alpha\}$ vector basis.

1. Write down the kinematic expressions for $\boldsymbol{r}_0^G$, $\boldsymbol{v}_0^G$, and $\boldsymbol{\omega}_{b0}$, as a function of $\alpha, \beta, \psi, \theta, \phi$ and their derivatives. Remember to first define any unit vectors that you may use in terms of $\{\boldsymbol{i}_0, \boldsymbol{j}_0, \boldsymbol{k}_0\}$ (**3 points**).
2. Express the three scalar constraint equations that relate the derivatives of $\alpha, \beta, \psi, \theta, \phi$. Is this constraint integrable? (**3 points**).
3. Identify and express mathematically all forces (indicating their point/line of application) and torques present in the problem. Introduce additional unknowns as needed. Reduce the force/torque system to a single resultant force and torque about G (**2 points**).
4. Identify and count the equations and unknowns in the problem. Is this problem closed? is it hyperstatic? Note that you do not need to write down the equations (**1 points**).
5. Is the mechanical energy of the ball conserved? Justify your answer. If it is conserved, write it down (**1 points**).