One Ring to rule them all

We want to analyze the motion of the ring with respect to an inertial reference frame $Oxyz$ (with unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$) as shown in the figure. We are going to consider the ring as a rigid body, with a body-fixed reference frame $Gx_By_Bz_B$ (with unit vectors $\boldsymbol{i_B}$, $\boldsymbol{j_B}$ and $\boldsymbol{k_B}$) attached to it. In the instant considered,

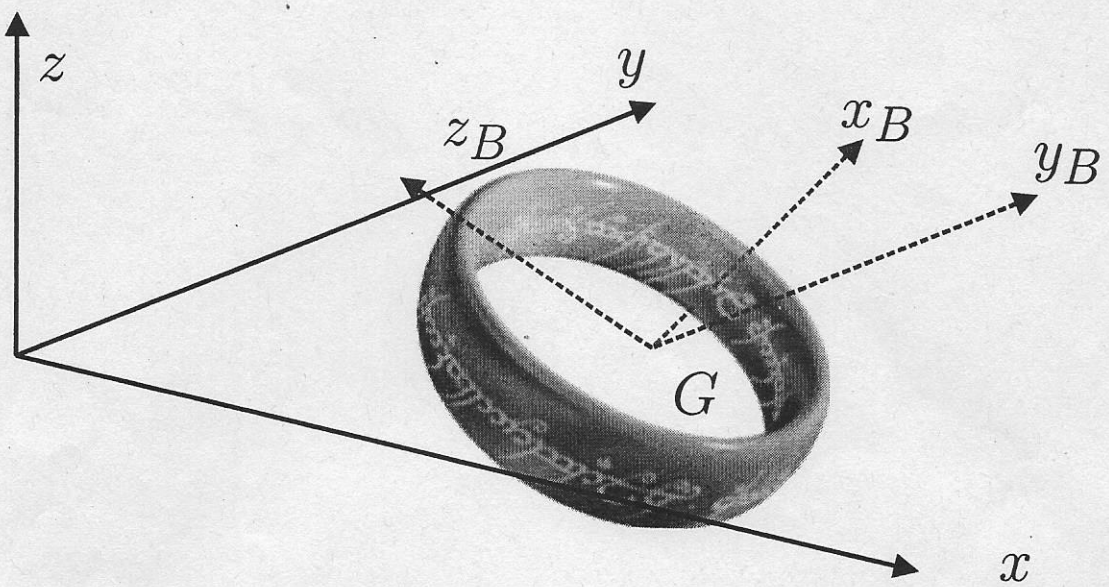
- the ring is rolling without slipping.
- the point of contact of the ring with the plane Oxy is C. The coordinates of C in the body-fixed reference frame are $\frac{R}{20}(-1, 0, -19)$.
- the velocity of G is $\boldsymbol{v}_G = \Omega R\left(-\boldsymbol{i} + \frac{\sqrt{5}}{2}\boldsymbol{j} - \frac{9}{10}\boldsymbol{k}\right)$
- the angular velocity fulfills the condition $\boldsymbol{\omega} \cdot \boldsymbol{i}_B = \boldsymbol{\omega} \cdot \boldsymbol{k}_B$
- the centre of mass G and the axes Gx_B and Gz_B are contained in the plane Oxz
- the angle between the axis Ox and Gx_B is 45°

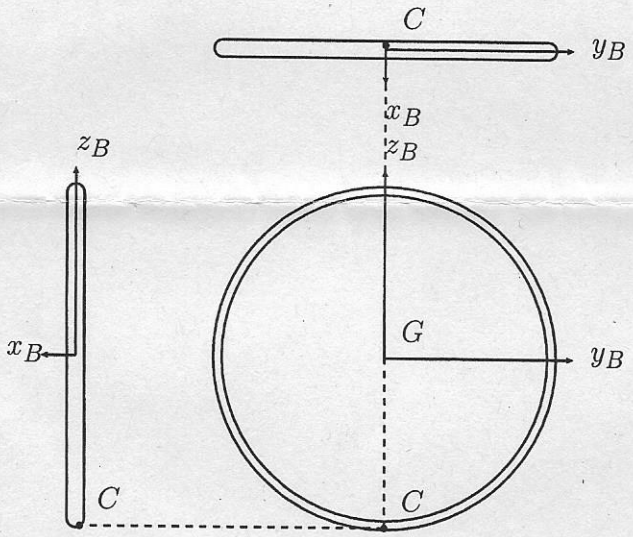
The mass of the ring is M and the inertia tensor with respect to the reference frame $Gx_By_Bz_B$ is approximately given by

$$\bar{\bar{I}}_G = MR^2 \begin{bmatrix} B & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{bmatrix}$$

where B, and D are considered as constants.

1. Compute the instantaneous axis of rotation
2. Compute the kinetic energy of the ring and the angular momentum of the ring with respect to G
3. Assuming Mordor is located at the origin and attracts the ring with a force proportional to the mass and to the distance (constant of proportionality β), compute the force and moment with respect to G due to the presence of Mordor.
4. If we know that $B = 2D$ and the moment of inertia with respect to G is $I_G = MR^2$, solve for $\bar{\bar{I}}_G$.


z
y
z_B
x_B
y_B
G
x


C
y_B
x_B
z_B
z_B
G
y_B
x_B
C
C