PARCIAL-1-MECHANICS-2022-2023.pdf

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Mecánica aplicada a la Ingeniería Aeroespacial

2º Grado en Ingeniería Aeroespacial

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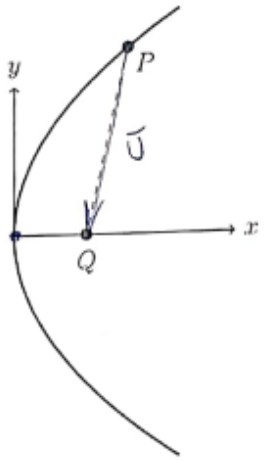
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A point particle P of mass m is constrained to move along a wire as shown in the figure. The wire has the shape of a parabola, given by

$$4\ell x - y^2 = 0,$$

where $\{x, y, z\}$ is the standard Cartesian coordinate system of the inertial reference frame S_0, and ℓ a known length constant. The particle experiences an attracting force $\boldsymbol{F}$ toward the focus Q of this parabola, which is located at $(x, y) = (\ell, 0)$. The magnitude of this force is proportional to mass of P and to the inverse of the square of the distance d between P and Q: $|\boldsymbol{F}| = mkd^{-2}$, where k is a known constant.

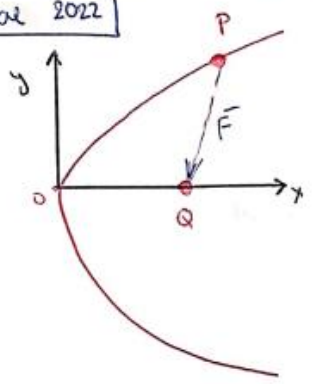


1. Give the expressions for the position, velocity, and acceleration of P as functions of y and its derivatives (**1.5 points**).
2. Give a tangent vector and an in-plane normal vector to the parabola, expressed as a function of y (**1 point**).
3. Find the unit vector $\boldsymbol{u}$ that goes in the direction from P to Q, and express the distance d, both as a function of y (**1.5 points**).
4. Identify and write down the general expression of all the forces acting on P, introducing new unknowns as needed (**1 point**).
5. Write down (a) a second-order differential equation for y that does not depend on any other unknowns, and (b) an equation for the magnitude of the constraint force that depends only on y and $\dot{y}$ (**1.5 points**).
6. Is the mechanical energy of P conserved? Is the angular momentum of P about Q always conserved? if one (or both) are conserved, write down the corresponding equation (**1.5 points**).
7. Using the solution to question 5, find the speed that P must have as a function of y for it to experience zero constraint force from the wire (**1 point**).
8. Using the solution to question 6, find the minimum speed of P at $(x, y) = (0, 0)$ that allows the particle to escape to infinity. (**1 point**).

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Parcial 2022



No gravity y no friction

$4\ell x - y^2 = 0 \rightarrow y = \sqrt{4\ell x} \rightarrow x = \frac{y^2}{4\ell}$

$S_0(x,y,t)$ → Inertial R.F

$\bar{Q} = (\ell, 0)$

$|\bar{F}| = \frac{mk}{d^2}$

a) Kinematics (P)

- $\bar{r}_0^P = x\vec{i} + y\vec{j} = \frac{y^2}{4\ell}\vec{i} + y\vec{j}$
- $\bar{v}_0^P = \dot{x}\vec{i} + \dot{y}\vec{j} = \frac{2y\dot{y}}{4\ell}\vec{i} + \dot{y}\vec{j} = \frac{y\dot{y}}{2\ell}\vec{i} + \dot{y}\vec{j}$
- $\bar{a}_0^P = \ddot{x}\vec{i} + \ddot{y}\vec{j} = \frac{y\ddot{y}}{2\ell}\vec{i} + \frac{\dot{y}^2}{2\ell}\vec{i} + \ddot{y}\vec{j}$

b)

b) Tangent vector, normal vector

Since we have a curve → 2 normal vectors, 1 tangent vector

- $\bar{N}_1 = N_1 \nabla f = N_1 (4\ell\vec{i} - 2y\vec{j})$
- $\bar{N}_2 = N_2 \vec{k}$

$\vec{t} = \vec{N}_1 \times \vec{N}_2$

$$\vec{t} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4\ell & -2y & 0 \\ 0 & 0 & 1 \end{vmatrix} = -2y\vec{i} - 4\ell\vec{j}$$

c) Unit vector $\bar{u}$ $\overline{PQ}$

$\overline{PQ} = \bar{u} = Q - P = \ell\vec{i} - \left[\frac{y^2}{4\ell}\vec{i} + y\vec{j}\right] = \left[\ell - \frac{y^2}{4\ell}\right]\vec{i} - y\vec{j}$

$|\overline{PQ}| = \sqrt{\left(\ell - \frac{y^2}{4\ell}\right)^2 + y^2}$

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d) Forces acting on P

→ No weight since there is no gravity

→ $\vec{F}_A = |Fa|\,\vec{u} = -\dfrac{mk\,\vec{PQ}}{|PQ|^3}$

→ $\vec{N} = N\,\vec{n} = N(4l\vec{i} - 2y\vec{j})$

e) 1. 2nd order ODE for y

$\boxed{\Sigma \vec{F} = m\vec{a}_{OP}}$

[i] $m\left[\dfrac{y\ddot{y}}{2l} + \dfrac{\dot{y}^2}{2l}\right] = \dfrac{mk}{d^3}\cdot\left[l - \dfrac{y^2}{4l}\right] + 4lN$

[j] $m\ddot{y} = -\dfrac{mk}{d^3}y - 2Ny$

$\boxed{(1)\,y + (2)\,2l}$

$ym\left[\dfrac{y\ddot{y}}{2l} + \dfrac{\dot{y}^2}{2l}\right] = \dfrac{mk}{d^3}y\left[l - \dfrac{y^2}{4l}\right] + 4lyN$

$2ml\ddot{y} = -2l\dfrac{mk}{d^3}y - 4lNy$

$$m\left[\frac{y^2\ddot{y}}{2l} + \frac{y\dot{y}^2}{2l} + 2l\ddot{y}\right] = \frac{mk}{d^3}y\left[l - \frac{y^2}{4l}\right] - 2l\frac{mk}{d^3}y$$

2. Equation of constrain

$(1)\,y - (2)\,2l$

$ym\left[\dfrac{y\ddot{y}}{2l} + \dfrac{\dot{y}^2}{2l}\right] = \dfrac{mk}{d^3}y\left[l - \dfrac{y^2}{4l}\right] + 4lyN$

$-2ml\ddot{y} = 2ly\dfrac{mk}{d^3} + 4lyN$

$$m\left[\frac{y^2\ddot{y}}{2l} + \frac{y\dot{y}^2}{2l} - 2l\ddot{y}\right] = \frac{mk}{d^3}y\left[l - \frac{y^2}{4l}\right] + 2yl\frac{mk}{d^3} + 2lyN$$

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$$m\left[\frac{y^2\ddot{y}}{2\ell} + \frac{y\dot{y}^2}{2\ell} - 2\ell\ddot{y}\right] - \frac{mk}{d^3}y\left[\ell - \frac{y^2}{4\ell}\right] - 2\ell y\frac{mk}{d^3} = 2\ell_y N$$

$$N = \frac{m\left[\frac{y^2\ddot{y}}{2\ell} + \frac{y\dot{y}^2}{2\ell} - 2\ell\ddot{y}\right] - \frac{mk}{d^3}y\left[\ell - \frac{y^2}{4\ell}\right] - 2\ell y\frac{mk}{d^3}}{2\ell_y}$$

6. Energy conserved

No fictious fora sra there is n rotation

- N = non-conservative but it does not exert work

$$\boxed{\vec{N} \perp v}$$

- $\vec{F}_A$ = conservative force

Therefore, energy will be conserved

7. $\boxed{\vec{N} = 0}$

Taking the equation in 4.2

$$m\left[\frac{y^2\ddot{y}}{2\ell} + \frac{y\dot{y}^2}{2\ell} + 2\ell\ddot{y}\right] = \frac{mk}{d^2}y\left[\ell - \frac{y^2}{4\ell}\right] - 2\ell y\frac{mk}{d^3}$$

$$\vdots$$

$$-\ddot{y}^2 = \frac{-2k\ell}{\left(\frac{y^2}{4\ell} + \ell\right)3}$$

8. Conservation of energy

$$\frac{1}{2}m|\vec{v}_0^{\,p}|^2 - \frac{km}{d} = E$$

$$|\vec{v}_0^{\,p}| = 0 \quad \text{at } \infty \quad \ldots \quad \boxed{V_{min} = \sqrt{\frac{2k}{\ell}}}$$

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