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2º Fundamentos de mecánica de vuelo

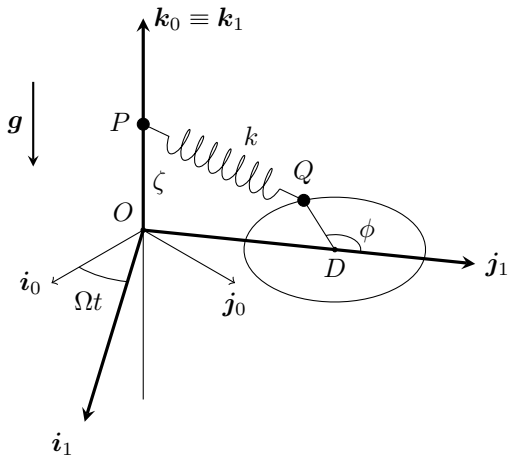
Grado en Ingeniería Aeroespacial

Escuela Politécnica Superior
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Two heavy particles P and Q, each of mass m, are connected with a spring of stiffness k as shown on the figure. An inertial reference frame S_0 is defined with origin O and vector basis $B_0 = \{\boldsymbol{i}_0, \boldsymbol{j}_0, \boldsymbol{k}_0\}$. Particle P slides without friction along the vertical axis, while particle Q slides without friction on a ring of radius R contained in the horizontal plane defined by O, $\boldsymbol{i}_0$ and $\boldsymbol{j}_0$. The center D of this ring is located a distance ℓ from O and rotates around the vertical axis of S_0 with a known constant angular rate Ω. A rotating reference frame S_1 is defined so that the $\boldsymbol{j}_1$ vector follows the position of D. We define the generalized coordinates ζ (vertical position of P) and ϕ (angle of Q on the circle from $\boldsymbol{j}_1$) as shown.



1. How many effective degrees of freedom does the system have? Define the vectors $\boldsymbol{i}_1$, $\boldsymbol{j}_1$ and $\boldsymbol{k}_1$ in terms of the vectors of B_0 (**1 point**).

   There are two effective degrees of freedom associated to ζ and ϕ. The vectors are:

   $$\boldsymbol{i}_1 = \cos(\Omega t)\boldsymbol{i}_0 + \sin(\Omega t)\boldsymbol{j}_0; \qquad \boldsymbol{j}_1 = -\sin(\Omega t)\boldsymbol{i}_0 + \cos(\Omega t)\boldsymbol{j}_0; \qquad \boldsymbol{k}_1 = \boldsymbol{k}_0.$$

2. Write down the general expression of the position, velocity, and acceleration of P and Q with respect to S_1 in terms of ζ, ϕ and their derivatives (**1.5 points**).

   $$\begin{aligned}
   \boldsymbol{r}_1^P &= \zeta\boldsymbol{k}_1; & \boldsymbol{r}_1^Q &= -R\sin\phi\boldsymbol{i}_1 + (\ell + R\cos\phi)\boldsymbol{j}_1;\\
   \boldsymbol{v}_1^P &= \dot{\zeta}\boldsymbol{k}_1; & \boldsymbol{v}_1^Q &= -R\dot{\phi}\cos\phi\boldsymbol{i}_1 - R\dot{\phi}\sin\phi\boldsymbol{j}_1;\\
   \boldsymbol{a}_1^P &= \ddot{\zeta}\boldsymbol{k}_1; & \boldsymbol{a}_1^Q &= -R(\ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi)\boldsymbol{i}_1 - R(\ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi)\boldsymbol{j}_1;
   \end{aligned}$$

3. Identify and write down the general expression of all the forces acting on P and Q (**2.5 points**).

   On P there is the weight, $\boldsymbol{W}_P = -mg\boldsymbol{k}_1$, the force of the spring, $\boldsymbol{F}_{S,P} = -kR\sin\phi\boldsymbol{i}_1 + k(\ell + R\cos\phi)\boldsymbol{j}_1 - k\zeta\boldsymbol{k}_1$, and the reaction of the vertical axis, $\boldsymbol{N}_P = N_1\boldsymbol{i}_1 + N_2\boldsymbol{j}_1$. On Q there is the weight, $\boldsymbol{W}_Q = -mg\boldsymbol{k}_1$, the force of the spring, $\boldsymbol{F}_{S,Q} = -\boldsymbol{F}_{S,P}$, and the reaction of the ring $\boldsymbol{N}_Q = N_3(-\sin\phi\boldsymbol{i}_1 + \cos\phi\boldsymbol{j}_1) + N_4\boldsymbol{k}_1$.

   Besides these applied and reaction forces, when doing the analysis in S_1 we will need to add the inertial forces. These can be given in this question or in the next. The only two nonzero inertia forces are the Coriolis force and the centrifugal force on Q: $\boldsymbol{F}_{I,Q} = 2mR\Omega\dot{\phi}\cos\phi\boldsymbol{j}_1 - 2mR\Omega\dot{\phi}\sin\phi\boldsymbol{i}_1 - mR\Omega^2\sin\phi\boldsymbol{i}_1 + m\Omega^2(\ell + R\cos\phi)\boldsymbol{j}_1$.

4. Using S_1 for your analysis, write down the scalar equations of motion of P and Q (**2.5 points**).

   S_1 is a non-inertial reference frame, hence we need to include inertial forces. Applying Newton's second

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law to each particle and projection the vector equations along we get:

$$\begin{aligned}
0 &= -kR\sin\phi + N_1 \\
0 &= k(\ell + R\cos\phi) + N_2 \\
m\ddot{\zeta} &= -mg - k\zeta \\
-mR(\ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi) &= kR\sin\phi - N_3\sin\phi - 2mR\Omega\dot{\phi}\sin\phi - mR\Omega^2\sin\phi \\
-mR(\ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi) &= -k(\ell + R\cos\phi) + N_3\cos\phi + 2mR\Omega\dot{\phi}\cos\phi + m\Omega^2(\ell + R\cos\phi) \\
0 &= -mg + k\zeta + N_4
\end{aligned}$$

5. Starting from these equations, write down a system of two differential equations for ζ and ϕ, that do not depend on any other unknowns of the problem (**1 point**).

   Equations 1, 2, and 6 are only good to determine N_1, N_2 and N_4. The equations we are looking for are 3, and a linear combination of 4 and 5:

$$\begin{aligned}
m\ddot{\zeta} &= -mg - k\zeta \\
-mR\ddot{\phi} &= -k\ell\sin\phi + m\ell\Omega^2\sin\phi
\end{aligned}$$

6. Find one conservation law of the problem (i.e., a first integral of motion) in reference frame S_1 (**1.5 points**).

   The mechanical energy of each particle alone is not conserved, but the mechanical energy of the system is. We can obtain this expression directly by adding T for each particle and the V of the spring, centrifugal force, and gravity, or by dot-multiplying the equations of motion by the velocity of P and Q and integrating:

$$\frac{1}{2}m(\dot{\zeta}^2 + R^2\dot{\phi}^2) + mg\zeta - \frac{1}{2}m\Omega^2(R^2 + \ell^2 + 2\ell R\cos\phi) + \frac{1}{2}k(R^2 + \ell^2 + 2\ell R\cos\phi + \zeta^2) = E$$

   Observe that the constant terms in the left hand side can be globbed into E to make the expression a bit simpler, without changing its meaning.

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