A heavy particle P is constrained to move along a circumference and it is attached to a spring as shown in the figure. The mass of the particle is m, and the radius of the circumference is a. Besides, one end of the spring is attached to a point of the y axis with coordinates $(0, \beta a)$ and the constant of the spring is equal to k.

1. Compute the potential of the forces acting on the particle as a function of the angle θ (shown in the figure).
2. Identify the equilibrium positions.

The particle departs from the point $(a, 0)$ of the circumference with an initial velocity $\mathbf{v}_0 = v_0\mathbf{j}$.

3. Reduce the motion to quadratures.
4. If $\beta = mg/(ka)$, obtain the position of the particle as a function of time.

Consider now the situation in which an additional force acts on the particle. A force is proportional to and in the same direction as the velocity. The constant of proportionality is $2m\omega$. Besides, the initial conditions are the same as before.

5. Obtain the position of the particle as a function of time.
6. Obtain the reaction of the circumference.

