4. We consider a point particle P of mass m, a spring of constant k and a damper of constant λ. The particle can only move in 1D, so that the equation of motion is

$$m\ddot{x} + \lambda\dot{x} + kx = 0.$$

The particle is released at $t = 0$ from $x = 0.01$ m with zero velocity.

(a) If $m = 2$ kg, $k = 5$ N/m and $\lambda = 6$ kg/s, explain qualitatively how the solution tends to $x \to 0$ as time advances. If this happens in the form of damped oscillations, determine the angular frequency of oscillation. **(0.5 points)**

[redacted]

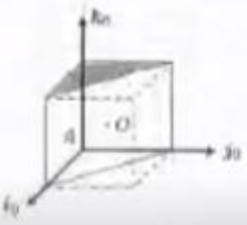
(b) If $m = 3$ kg, $k = 9$ N/m and $\lambda = 12$ kg/s, explain qualitatively how the solution tends to $x \to 0$ as time advances. If this happens in the form of damped oscillations, determine the angular frequency of oscillation. **(0.5 points)**

[redacted]

5. Of an air vehicle we know the position and velocity of three points at a given instant of time: A, located at $r_0^A = 3i + 2k$, has $v_0^A = 8i + 6j - 6k$. Point B, located at $r_0^B = j + 5k$, has $v_0^B = 16i - 3j + 5k$. And point C, located at $r_0^C = -i - j - 2k$, has $v_0^C = -3i + 10j + 4k$. Does this data allow us to assure that the vehicle moves as a rigid body? Find a point and the direction of the instantaneous axis of rotation and slip at that instant, and compute $v_\parallel$ **(1.5 points)**

[redacted]

6. The tensor of inertia of a homogeneous cube of side length ℓ and mass m about its center O is $\bar{I}_O = m\ell^2\bar{U}/6$, where $\bar{U}$ is the identity tensor. The cube is cut diagonally in two halves as displayed below. Find the position vector of the center of mass G of the shown half with respect to the given reference frame. Compute the tensor of inertia about G and express it in the given basis **(1.5 points)**.



7. An airplane has a body fixed reference frame S_b with origin at its center of mass G and vector basis $B_b = \{i_b, j_b, k_b\}$. At a given instant of time, its attitude with respect to a reference frame S_0 with basis $B_0 = \{i_0, j_0, k_0\}$ is defined in terms of 2-1-3 Euler angles $\theta = -90$ deg, $\psi = 180$ deg, and $\varphi = 90$ deg (**1 point**).

(a) Express the vectors of B_b in B_0.

$\bar{i}_b = -\bar{j}_0$ ✓

$\bar{j}_b = -\bar{k}_0$

$\bar{k}_b = \bar{i}_0$

(b) Are there other sets of 2-1-3 Euler angles that result in exactly the same attitude? Justify your answer.

8. Identify which of the following statements are TRUE or FALSE about the torque-free motion of a rigid body and provide a brief justification of your answer. In the following, S_0 is an inertial reference frame with origin at the center of mass G of the body, and S_b is a body-fixed reference frame with origin and G and all axes along principal directions of inertia (**1 point**).

(a) The angular momentum $\boldsymbol{H}_{G0}$ is a constant vector in S_0 but the angular momentum $\boldsymbol{H}_{Gb}$ is not a constant vector in S_b.

(b) If the body is axisymmetric, then its motion is a combination of precession and nutation about the symmetry axis.

(c) If the body is axisymmetric, oblate, and the nutation angle is smaller than 90 deg, then the spin and the precession rates have different sign.

(d) In the case of a purely asymetric body, we can assert that the rotation about the major axis of inertia is always stable.