FLUID MECHANICS I **12-12-2025**
SHORT QUESTIONS 1 (10 POINTS) 30%. **Time: 25 minutes**

NAME:..... **GRADE:.....**

Dimensional analysis and physical similarity

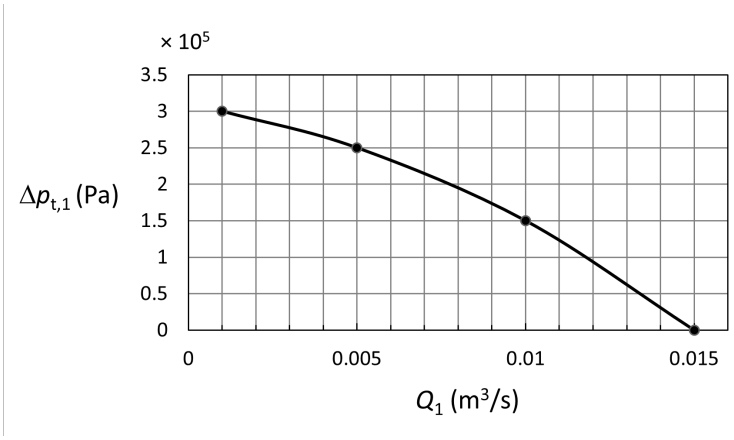
The following dependence relationship applies for the total pressure increment, Δp_t, that a centrifugal pump imparts to a flow rate Q of a liquid of density ρ and dynamic viscosity μ.

$$\Delta p_\mathrm{t} = f(D, \omega, \rho, \mu, Q),$$

where D and ω are the diameter and the angular velocity of the rotor. This applies for geometrically similar pumps.

1. Use dimensional analysis techniques to reduce the parametric dependence relation for Δp_t.
2. Simplify the relation for cases with negligible viscous effects. Keep this approximation for the following questions, always justifying its validity.

A centrifugal pump of $D_1 = 0.2$ m, rotating at $\omega_1 = 120$ rad/s and working with water (density $\rho = 10^3$ kg/m^3 and viscosity $\mu = 10^{-3}$ Pa s) gives the following results



3. Obtain the curve $\Delta p_{\mathrm{t},2}$ vs ω_2 for the same pump ($D_2 = D_1$) working with $Q_2 = 0.005$ m^3/s.
4. Obtain the curve $\Delta p_{\mathrm{t},3}$ vs Q_3 for a geometrically similar pump of diameter $D_3 = 0.4$ m rotating at $\omega_3 = 90$ rad/s.

$$\Delta p_t = f(D, w, \rho, \mu, Q)$$

	Δp_t	D	w	ρ	μ	Q
M	1	0	0	1	1	0
L	-1	1	0	-3	-1	3
T	-2	0	-1	0	-1	-1

$n = 5$, $k = 3$ $\Rightarrow$ $n - k = 2$

$\Pi_0 = f(\Pi_1, \Pi_2)$

pick ρ, w, D.

Check that these are dimensionally independent

$$\begin{vmatrix} \rho & w & D \\ 1 & 0 & 0 \\ -3 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} = 1 \neq 0$$

Then,

$$\frac{\Delta p_t}{\rho w^2 D^2} = f\left(\frac{\rho w D^2}{\mu}, \frac{Q}{wD^3}\right)$$

or pick ρ, Q, D

$$\begin{vmatrix} \rho & Q & D \\ 1 & 0 & 0 \\ -3 & 3 & 1 \\ 0 & -1 & 0 \end{vmatrix} = 1 \neq 0$$

Then

$$\frac{\Delta p_t D^4}{\rho Q^2} = f\left(\frac{\rho Q}{\mu D}, \frac{Q}{wD^3}\right)$$

↳ both options are valid and will give the same results in the similarity problem below

To define the Reynolds number, the characteristic velocity inside the pump is best represented by wD

$\rightsquigarrow$ $Re = \frac{\rho w D^2}{\mu}$

If viscous effects are negligible $\frac{\rho \omega D^2}{\mu} \gg 1$

$$\frac{\Delta p_t}{\rho \omega^2 D^2} = f\left(\frac{Q}{\omega D^3}\right) \qquad \frac{\Delta p_t D^4}{\rho Q^2} = f\left(\frac{Q}{\omega D^3}\right)$$

physical similarity

if $\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}$ then $\frac{\Delta p_{t1}}{\rho_1 \omega_1^2 D_1^2} = \frac{\Delta p_{t2}}{\rho_2 \omega_2^2 D_2^2}$

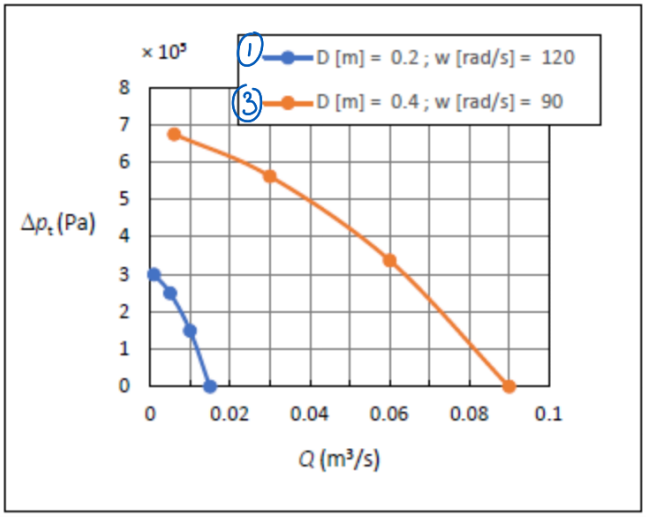
or $\frac{\Delta p_{t1} D_1^4}{\rho_1 Q_1^2} = \frac{\Delta p_{t2} D_2^4}{\rho_2 Q_2^2}$

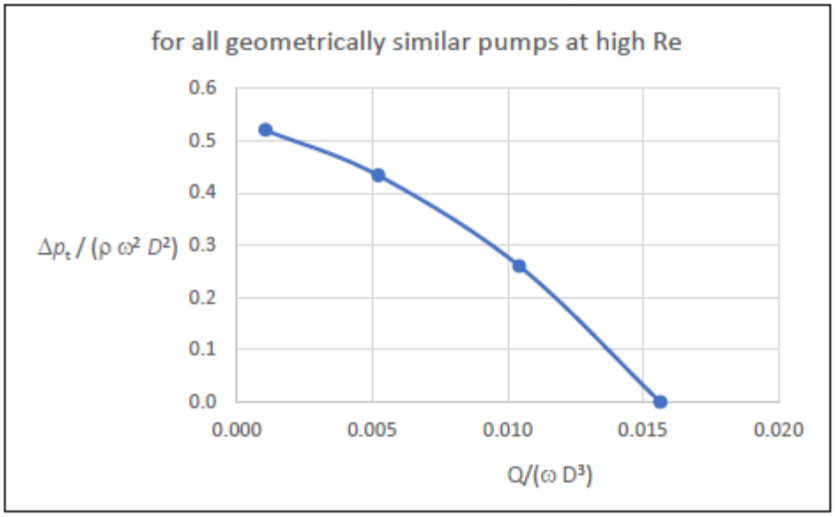
$\rho = 10^3 \frac{kg}{m^3}$, $\mu = 10^{-3} \frac{kg}{ms}$ } water in all cases

$$\omega_2 = \omega_1 \frac{Q_2}{Q_1}\left(\frac{D_1}{D_2}\right)^3 \qquad \Delta p_{t2} = \Delta p_{t1} \frac{\rho_2}{\rho_1} \frac{\omega_2^2}{\omega_1^2} \frac{D_2^2}{D_1^2}$$

$$Q_3 = Q_1 \frac{\omega_3}{\omega_1} \frac{D_3^3}{D_1^3} \qquad \Delta p_{t3} = \Delta p_{t1} \frac{\rho_3}{\rho_1} \frac{\omega_3^2}{\omega_1} \frac{D_3^2}{D_1^2}$$

① D [m] = 0.2; ρ [kg/m3] = 1000; μ [Pa s] = 1.00E-03; ω [rad/s] = 120; Re = ρ ω D²/μ = 4.80E+06		all geometrically similar pumps working at high Reynolds numbers		② D [m] = 0.2; ρ [kg/m3] = 1000; μ [Pa s] = 1.00E-03; Q [m³/s] = 0.005			③ D [m] = 0.4; ρ [kg/m3] = 1000; μ [Pa s] = 1.00E-03; ω [rad/s] = 90; Re = ρ ω D²/μ = 1.44E+07	
Q [m³/s]	Δp_t [Pa]	Q/(ω D³)	Δp_t / (ρ ω² D²)	ω [rad/s]	Δp_t [Pa]	Re	Q [m³/s]	Δp_t [Pa]
0.001	300000	0.00104	0.521	600	7500000	2.40E+07	0.006	675000
0.005	250000	0.00521	0.434	120	250000	4.80E+06	0.030	562500
0.01	150000	0.01042	0.260	60	37500	2.40E+06	0.060	337500
0.015	0	0.01563	0.000	40	0	1.60E+06	0.090	0





FLUID MECHANICS I **12-12-2025**
PROBLEM 1 (10 POINTS) 70%. **Time: 65 minutes**

NAME:..... **GRADE:.....**

Channel with 2 liquids

The sketch shows 2 large reservoirs with the same level H of fluids 1 and 2, with densities and viscosities $\rho_1 > \rho_2$ and $\mu_1 > \mu_2$, respectively. The tanks are connected by a channel of length L and height h. The top of the channel is made up of a plate of mass M, hinged at the point O and simply supported at the other end, as shown in the sketch. The channel is initially filled with liquids 1 and 2, at equal lengths $L/2$. Due to gravity g, the liquid in tank 1 will flow towards tank 2. Known magnitudes: ρ_1, μ_1, ρ_2, μ_2, L, H, h, g, p_a

1. If the flow is dominated by viscosity, estimate the longitudinal velocity V_L, the pressure variation $\Delta_L P$, and the characteristic time t_0. Give the criteria for the flow to be dominated by viscosity, based on known magnitudes. Use may be made of the properties of fluid 1 to scale the results.

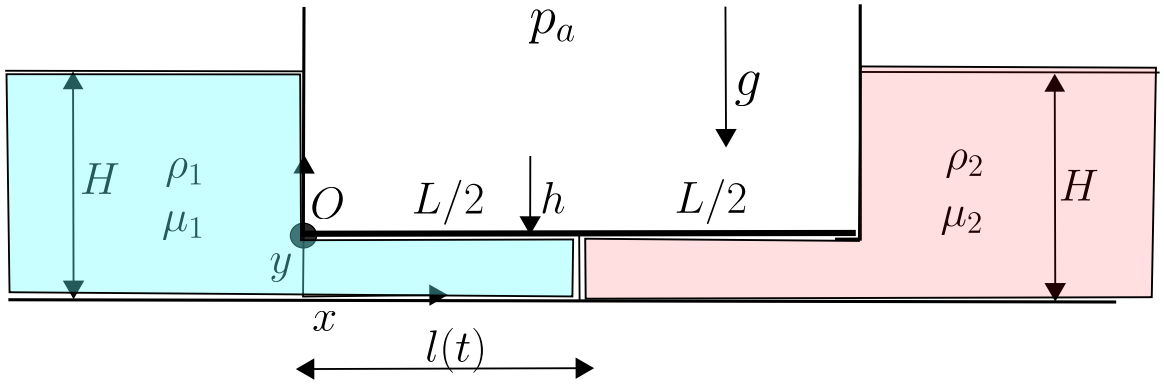
2. Determine the flow rate Q as a function of the position l of the interphase between fluids 1 and 2

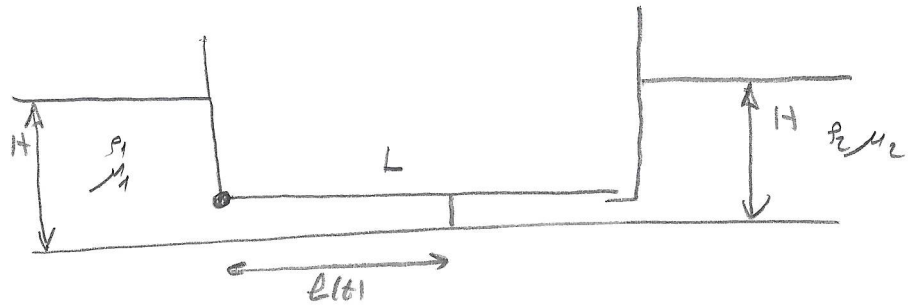
3. Obtain the position of the interface $l(t)$ as a function of time and determine the time t_f required by fluid 1 to reach tank 2.

In the following, we will analyze the case $\rho_2/\rho_1 \ll 1,\ \mu_2/\mu_1 \ll 1$.

4. Compute the time t_f in this limit. Determine the net force exerted on the hinged top plate as a function of l.

5. Obtain the moment exerted on the top plate as a function of l and give the minimum mass M to prevent the plate from rotating at any time.





① Momentum x: $\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} + \mu\frac{\partial^2 u}{\partial x^2}$

$\frac{\rho V_L}{t_0}$ $\frac{\rho V_L^2}{L}$ $\frac{\Delta L P}{L}$ $\frac{\mu V_L}{h^2} \gg \frac{\mu V_L}{L^2}$

$t_0 \sim \frac{L}{V_L} \Rightarrow \frac{\rho V_L^2}{L}$

$\boxed{V_L \sim \frac{\Delta L P}{L}\frac{h^2}{\mu_1} \sim \frac{\rho_1 g H h^2}{\mu_1 \cdot L}}$

$\boxed{\Delta L P \sim \rho_1 g H}$

$\boxed{t_0 \sim \frac{\mu_1 L^2}{\rho_1 g H h^2}}$

$\boxed{\frac{\text{Convection}}{\text{Viscous}} \sim \frac{\frac{\rho V_L^2}{L}}{\frac{\mu V_L}{h^2}} \sim \frac{\rho V_L h}{\mu_1}\cdot\frac{h}{L} \sim \frac{\rho_1^2 g H h^4}{\mu_1^2 L^2} \ll 1}$

② Dominated by viscosity: $0 = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2}$

$y=0: u=0$
$y=h: u=0$

$\boxed{u = -\frac{1}{2\mu}\frac{\partial P}{\partial x}\, y(h-y)}$

$\boxed{Q = \int_0^h u\,dy = \frac{-h^3}{12\mu_i}\left(\frac{\partial P}{\partial x}\right)_i}$ valid for both fluids. $i = 1, 2$ $\quad Q_1 = Q_2 = Q$

Q independent of x $\Rightarrow$ LIQUID 1: $P_a + \rho_1 g H - P_1(x) = \frac{12\mu_1 Q}{h^3}x$

LIQUID 2: $P_2(x) - (P_a + \rho_2 g H) = \frac{12\mu_2 Q}{h^3}(L-x)$

AT INTERFACE: $x = \ell(t): P_1(\ell) = P_2(\ell)$

$P_1(\ell) = P_a + \rho_1 g H - \frac{12\mu_1 Q}{h^3}\ell$

$P_2(\ell) = P_a + \rho_2 g H + \frac{12\mu_2 Q}{h^3}(L-\ell)$

$\boxed{(\rho_1-\rho_2)gH = \frac{12\mu_1 Q\ell}{h^3} + \frac{12\mu_2 Q(L-\ell)}{h^3}}$

Therefore: $\boxed{Q(\ell) = \frac{(\rho_1-\rho_2)gH h^3}{12\mu_1\ell + 12\mu_2(L-\ell)}}$

③ Take into account: $Q(t) = h\cdot\frac{d\ell}{dt} = \frac{(\rho_1-\rho_2)gH h^3}{12\mu_2 L + 12(\mu_1-\mu_2)\ell}$

$\ell(0) = \frac{L}{2}$

$\boxed{\frac{(\rho_1-\rho_2)}{12}gH h^2 t = \mu_2 L\left(\ell - \frac{L}{2}\right) + (\mu_1-\mu_2)\left(\frac{\ell^2}{2} - \frac{L^2}{8}\right)}$

AND: $\frac{(\rho_1-\rho_2)}{12}gH h^2 t_f = \mu_2\frac{L^2}{2} + (\mu_1-\mu_2)\frac{3}{8}L^2$

$\boxed{t_f = \frac{3}{4}\frac{L^2\mu_1}{\rho_1 g H h^2}\frac{(3+\mu_2/\mu_1)}{1-\frac{\rho_2}{\rho_1}} \sim t_0}$

(4) $\frac{\rho_2}{\rho_1} \ll 1$; $\frac{\mu_2}{\mu_1} \ll 1$ $\Rightarrow$ $\boxed{t_f \simeq \frac{3L^2\mu_1}{4\rho_1 g H h^2}}$

Now $P_2 \approx P_a$

$$P_1(x) = P_a + \rho_1 g H - \frac{12\mu_1 Q}{h^3} x$$

$$\boxed{F_y = \int_0^l (P - P_a)\, dx = \rho_1 g H l - \frac{12\mu_1 Q}{h^3}\frac{l^2}{2}}$$

$$F_x = \int_0^l -\mu \left.\frac{\partial u}{\partial y}\right|_{y=h} dx = \int_0^l -\mu \left(-\frac{1}{2\mu}\frac{\partial P}{\partial x}(h-2y)\right)_{y=h} dl = \int_0^l \frac{-h}{2}\frac{\partial P}{\partial x}\, dl$$

$$\boxed{F_x = \frac{h}{2}\cdot\frac{12\mu_1 Q}{h^3} l = \frac{6\mu_1 Q l}{h^2}}$$

(5) $M_0 = \int_0^l (P - P_a)\, x\, dx \bar{e}_z = \rho_1 g H \frac{l^2}{6}\bar{e}_z$ $\rightarrow$ CRITICAL $l = L$

$M_W = -Mg\frac{L}{2}\bar{e}_z$

NO ROTATION $\Rightarrow$ $Mg\frac{L}{2} > \rho_1 g \frac{L^2}{6} H \Rightarrow \boxed{M > \frac{\rho_1 HL}{3}}$