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UC3MEXAMS

Mecánica de Fluidos I

2º Grado en Ingeniería Aeroespacial

Escuela Politécnica Superior. Campus de Leganés
Universidad Carlos III de Madrid

Accede al documento original

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FLUID MECHANICS I **12-12-2022**
PROBLEM 1 (16 POINTS). **Time: 75 minutes**

NAME:..... **GRADE:.....**

The liquid of density ρ and viscosity μ flows out a very large tank through a vertical cylindrical tube of radius R_T and length $L \gg R_T$ under the action of the gravity field. As shown in the attached figure, inside the tube there is a concentric thread of radius $R_0 < R_T$, which extends well below the lower end of the tube. It is requested to study the flow in the pipe during the stationary discharge of the liquid from the tank to determine the values of the volumetric flow rate Q. With this goal in mind,

1. Estimate the characteristic longitudinal velocity V_L and give a criterion for the movement of the liquid to be dominated by viscosity. Assume hereafter that this condition holds.

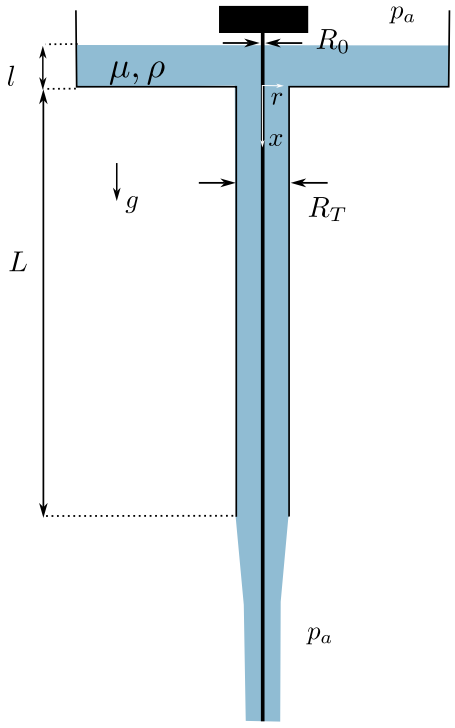
2. Using dimensional analysis, write the the volumetric flow rate Q in terms of the minimum number of non-dimensional parameters. Reduce the parametric dependence assuming the limit $l/L \ll 1$.

3. Determine the velocity profile of the liquid inside the tube. Please note that the liquid height in the tank can be considered constant and negligible compared to L, and that the pressure is atmospheric in the region of the jet.

4. Obtain the volumetric flow rate Q and write it in the form $Q = \left(P_l R_T^4/\mu\right) C(R_0/R_T)$, where $P_l = -dP/dx$ is the gradient of reduced pressure $P = p - \rho g x$ and $C(R_0/R_T)$ is a quadrature that is only a function of the parameter R_0/R_T. Do NOT evaluate the quadrature and assume hereafter that the value of C is known once the ratio R_0/R_T is given.

5. Obtain the pressure field p in the tube as a function of Q if the atmospheric pressure p_a is known.

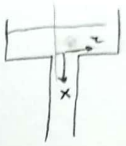
6. Obtain an explicit expression for the volumetric flow rate in terms of C and the rest of parameters of the problem, $(L, R_0, R_T, \rho, \mu, g)$. Write the expression in non-dimensional form using the results obtained in item 2.



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Problem 1



1) $\rho g \sim \frac{\mu \cdot U_c}{R_T^2} \Rightarrow U_c \sim \frac{\rho g R_T^2}{\mu}$

Flow dominated by viscosity when $\frac{\rho \cdot U_c^2/L}{\mu \cdot U_c/R_T^2} \sim \frac{\rho U_c R_T}{\mu} \frac{R_T}{L} \ll 1$

$$\boxed{\frac{\rho^2 g R_T^4}{\mu^2 L} \ll 1}$$

2) $Q = Q(\rho, \mu, l, L, R_0, R_T, g)$

Parameters with independent dimensions: μ, g, R_T

$$\frac{Q}{(g R_T^5)^{1/2}} = \tilde{Q}\left(\frac{\rho}{\mu}\cdot(g R_T^3)^{1/2}, \frac{R_0}{R_T}, \frac{l}{R_T}, \frac{L}{R_T}\right)$$

Actually, previous expression can be written as l/L

$$\frac{Q}{(g R_T^5)^{1/2}} = \tilde{Q}\left(\frac{\rho}{\mu}(g R_T^3)^{1/2}, \frac{R_0}{R_T}, \frac{l}{L}\right) \Rightarrow \boxed{\frac{Q}{(g R_T^5)^{1/2}} = \tilde{Q}\left(\frac{\rho}{\mu}(g R_T^3)^{1/2}, \frac{R_0}{R_T}\right)}$$

3) $0 = P_l + \mu \frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\cdot\frac{\partial v_x}{\partial r}\right)$ $\qquad P_l = -\frac{d}{dx}(p - \rho g x) = -\frac{dp}{dx}$

$v_x(R_0) = 0$, $v_x(R_T) = 0$ $\qquad p = p - \rho g x$

$$v_x(r) = C_2 + C_1 \ln r - \frac{P_l}{4\mu} r^2$$

Boundary conditions: $C_1 = -\frac{P_l R_T^2}{4\mu}\frac{1-(R_0/R_T)^2}{\ln(R_0/R_T)}$, $C_2 = -C_1 \ln R_T + \frac{P_l R_T^2}{4\mu}$

$$v_x(r) = \frac{P_l \cdot R_T^2}{4\mu}\left[\underbrace{\left(1-\left(\frac{r}{R_T}\right)^2\right) - \left(1-\left(\frac{R_0}{R_T}\right)^2\right)\frac{\ln\left(\frac{r}{R_T}\right)}{\ln\left(\frac{R_0}{R_T}\right)}}_{\tilde{v}\left(\frac{r}{R_T}\right)}\right] = \frac{P_l \cdot R_T^2}{4\mu}\cdot\tilde{v}\left(\frac{r}{R_T}\right)$$

4) $Q = \int_{R_0}^{R_T} v_x(r)\, d\sigma = \int_{R_0}^{R_T} \frac{P_l R_T^2}{4\mu}\cdot\tilde{v}\left(r/R_T\right)\cdot 2\pi\cdot r\cdot dr = \frac{P_l R_T^4}{\mu}\cdot\frac{\pi}{2}\int_{\frac{R_0}{R_T}}^{1}\frac{r}{R_T}\cdot\tilde{v}\left(\frac{r}{R_T}\right)\cdot d\left(\frac{r}{R_T}\right)$

$$\boxed{Q = \frac{P_l R_T^4}{\mu}\cdot G\left(\frac{R_0}{R_T}\right)} \qquad G = \frac{\pi}{2}\int_{\frac{R_0}{R_T}}^{1}\frac{r}{R_T}\tilde{v}\left(\frac{r}{R_T}\right)\cdot d\left(\frac{r}{R_T}\right)$$

5) $-\frac{dp}{dx} = \frac{\mu\cdot Q}{R_T^4\cdot G} \Rightarrow p = -\frac{\mu\cdot Q}{R_T^4\cdot G}\cdot x + K$

Imposing boundary conditions for pressure:

$x = 0$: $p = p_a + \rho g l = K$

$x = L$: $p = p_a - \rho g L = -\frac{\mu Q}{R_T^4 G} + p_a + \rho g l$

$$\Downarrow$$

$$\boxed{Q = \rho g \frac{(L+l)}{L}\cdot\frac{R_T^4\cdot G}{\mu}}$$

6) Using the results of item 2) $\qquad l/L \ll 1$

$$\boxed{\frac{Q}{(g R_T^5)^{1/2}} = G\cdot\frac{\rho}{\mu}\cdot(g R_T^3)^{1/2}\cdot\left(1-\frac{l}{L}\right) \simeq G\left(\frac{R_0}{R_T}\right)\cdot\frac{\rho}{\mu}(g R_T^3)^{1/2}}$$

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FLUID MECHANICS I **12-12-2022**
EXERCISE 2 (4 POINTS). **Time: 20 minutes**

NAME:..... **GRADE:.....**

In a planar flow, as sketched below, a perfect liquid, initially at rest, of density ρ and viscosity μ is contained between two infinitely long walls, separated a distance h. At $t = 0$ the wall starts to move with an alternative velocity $V\cos(\omega t)$ in the horizontal direction while the upper wall remains at rest. If the pressure gradient is zero,

1. Show that the flow is described by the ODE

$$\rho\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial y^2}$$

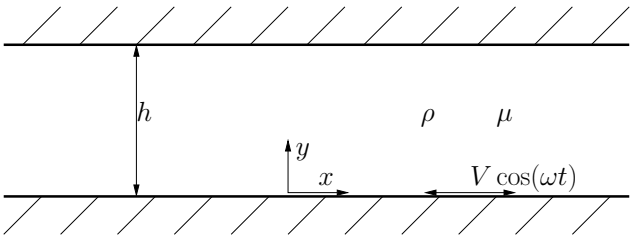
   Write the necessary boundary and initial conditions and complete the mathematical formulations of the problem

2. Estimate the thickness δ of the liquid affected by the movement of the lower wall, in terms of the data.

3. Non-dimensionalize the problem using as scales h for the length, ω^{-1} for the time and V for the velocity. Identify the only non-dimensional parameter Δ and show its relation with δ.

4. Consider the case $h \ll \delta$. Simplify the problem adequately find the solution.

Note 1:



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Solution:

As in page 79 of the Course Notes:

1. Because of the geometry, $\partial u/\partial x = 0$, and then from continuity and B.C. at walls $v = 0$.

   The x-momentum equation, using $\nabla P = 0$, with boundary conditions becomes

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2}$$
$$t = 0 : u = 0$$
$$y = 0 : u = V \cos(\omega t)$$
$$y = h : u = 0$$

2. $$\rho V \omega \sim \mu V/\delta^2 \Rightarrow \delta \sim \sqrt{\frac{\mu}{\rho\omega}}$$

3. $U = u/V$, $\tau = \omega t$, $\eta = y/h$ Therefore,

$$\frac{\partial U}{\partial \tau} = \Delta^2 \frac{\partial^2 U}{\partial \eta^2}$$
$$\tau = 0 : U = 0$$
$$\eta = 0 : U = \cos(\tau)$$
$$\eta = 1 : U = 0$$

   with $\Delta = \delta/h$.

4. If $h \ll \delta \Rightarrow \Delta \to \infty$, we obtain the quasi-steady problem $\partial^2 U/\partial \eta^2 \simeq 0$, with the B.C. above. The solution is $U = (1 - \eta)\cos(\tau)$, which means the flow quickly adapts to the velocity of the lower wall.

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