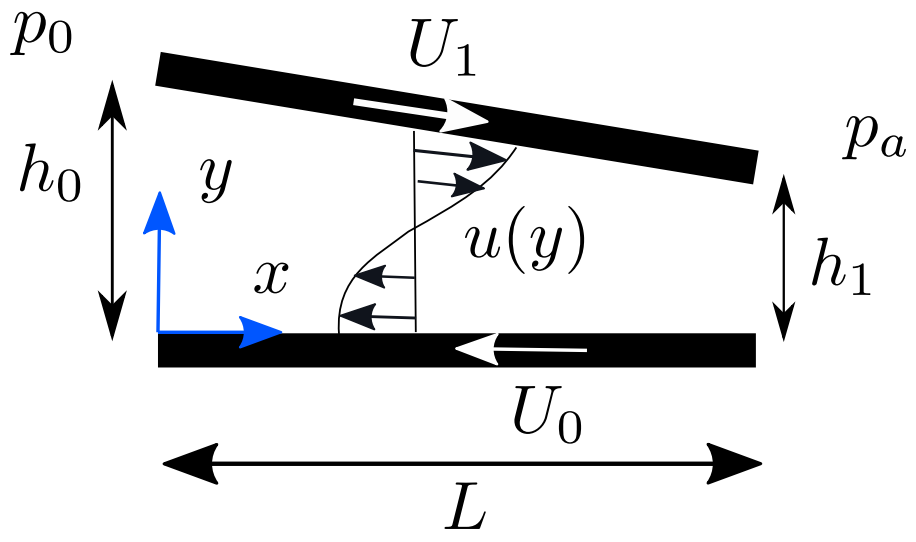
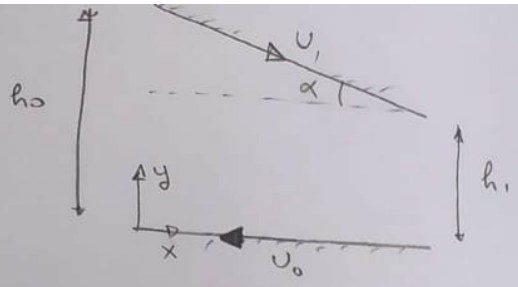
FLUID MECHANICS I **13-12-2021**
PROBLEM 1 (15 POINTS). **Time: 70 minutes**

NAME:..... **GRADE:.....**

The sketch below models the flow induced by the two moving walls that form a two-dimensional, variable-height channel of length $L \gg h$. The maximum height of the channel is h_0 and the minimum is h_1, as indicated in the figure. Both the upper and lower wall move with velocities U_1 and U_0, respectively. Assuming that the flow is dominated by viscosity and that $\Delta p = p_0 - p_a$ is known, it is asked

1. Obtain the velocity field $u(y)$ in terms of the wall velocities U_1 and U_0 and $P_l = -dp/dx$.
2. Determine the value of the volumetric flux Q in terms of the wall velocities U_1, U_0 and the rest of parameters of the problem.
3. Obtain the pressure distribution inside the channel $p(x) - p_0$.
4. Determine the relationship between U_0 and U_1 to induce a pressure loss Δp in the channel.





$$\tan\alpha \sim \alpha = \frac{h_0 - h_1}{L}$$

$$h = h_0 - (h_0 - h_1)\cdot\frac{x}{L}$$

$$\Delta p = p_0 - p_a$$

1) $$0 = -\frac{dp}{dx} + \mu\cdot\frac{d^2u}{dy^2}$$

$$u(0) = -U_0$$

$$u(h) = U_1\cos\alpha \simeq U_1$$

$$\Big\} \quad u(y) = -\frac{\rho_l\cdot y^2}{2\mu} + \left(\frac{U_1+U_0}{h} + \frac{\rho_l\cdot h}{2\mu}\right)y - U_0$$

2) $$Q = \int_0^h u\,dy = -\frac{\rho_l\cdot h^3}{6\mu} + \left(\frac{U_1+U_0}{h} + \frac{\rho_l h}{2\mu}\right)\frac{h^2}{2} - U_0\cdot h$$

$$\boxed{Q = \frac{\rho_l\cdot h^3}{12\mu} + \frac{U_1 - U_0}{2}h} \quad (1)$$

Using continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ $\rightarrow$ $\int_0^h \frac{\partial u}{\partial x}\,dy + v\Big|_0^h = 0$

$$\frac{d}{dx}\int_0^h u\,dy - U_1\cos\alpha\frac{dh}{dx} - U_1\cdot\alpha = 0$$

$$\frac{dQ}{dx} + U_1\alpha - U_1\alpha = 0 \rightarrow \frac{dQ}{dx} = 0 \rightarrow \boxed{Q = Q_0}$$

3) From (1) $\Rightarrow$ $$\frac{dp}{dx} = -\left(Q - \frac{U_1 - U_0}{2}h\right)\cdot\frac{12\mu}{h^3}$$

$$\frac{dp}{dx} = -\frac{12\mu\cdot Q_0}{h_0^3\left[1 - \frac{h_0-h_1}{h_0}\cdot\frac{x}{L}\right]^3} + \frac{6\mu(U_1-U_0)}{h_0^2\left[1 - \frac{h_0-h_1}{h_0}\cdot\frac{x}{L}\right]^2}$$

$$p(x) = \frac{-6\mu Q_0\cdot L}{h_0^2\cdot[h_0 - h_1]}\cdot\frac{1}{\left[1 - \frac{h_0-h_1}{h_0}\cdot\frac{x}{L}\right]^2} + \frac{6\mu\cdot(U_1-U_0)\cdot L}{h_0(h_0-h_1)}\cdot\frac{1}{1 - \frac{h_0-h_1}{h_0}\cdot\frac{x}{L}} + C$$

$x = 0 \quad p(0) = p_0 \Rightarrow$

$$C = p_0 + \frac{6\mu Q_0 L}{h_0^2(h_0-h_1)} - \frac{6\mu\cdot(U_1-U_0)\cdot L}{h_0(h_0-h_1)}$$

$$p(x) - p_0 = \frac{6\mu Q_0\cdot L}{h_0^2[h_0-h_1]}\left[1 - \frac{1}{\left[1 - \frac{h_0-h_1}{h_0}\frac{x}{L}\right]^2}\right] + \frac{6\mu(U_1-U_0)L}{h_0^2}\cdot\frac{x/L}{1 - \frac{h_0-h_1}{h_0}\cdot\frac{x}{L}}$$

4) $Q_0 = 0$, $x = L$, $p = p_a$ $\Big\}$ $\rightarrow$ $$p_a - p_0 = \frac{6\mu\cdot(U_1-U_0)L}{h_0^2}\cdot\frac{1}{h_1/h_0}$$

$$\boxed{U_1 = \frac{-\Delta p\cdot h_1 h_0}{6\mu L} + U_0}$$

FLUID MECHANICS I **13-12-2021**
EXERCISE 2 (5 POINTS). **Time: 25 minutes**

NAME:..... **GRADE:.....**

The figure shows a channel of length $2L$ and a sudden contraction in the middle, where the channel height changes from $2h$ to h. In the case of a steady flow of a fluid with density ρ and viscosity μ, with inlet velocity U,

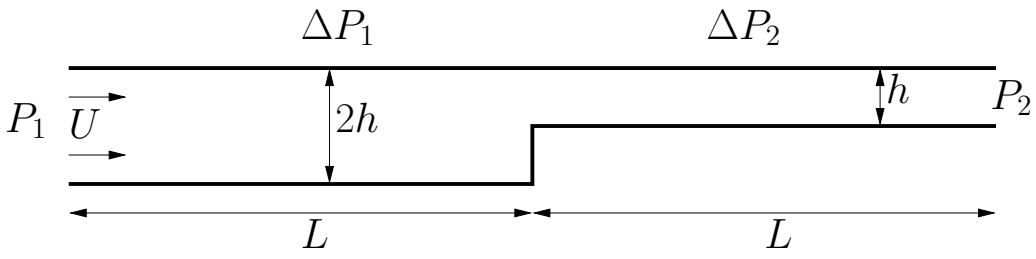
1. Find the parametric dependence of the reduced pressure variation $\Delta P = P_1 - P_2$. Use the Pi theorem to reduce the number of parameters.

2. Simplify the result for $Re = \rho U h/\mu \ll 1$. Do the same when $h/L \ll 1$.

3. Give the conditions for applicability of the theory of slender flows dominated by viscosity to be applicable in the configuration under study.

4. Estimate the orders of magnitude of ΔP_1 and ΔP_2 using the viscous flow theory, in each of the pieces of the pipe

5. Show that the pressure variation $\Delta_E P$, in the region of the channel contraction, is negligible compared with ΔP_1 and ΔP_2.





$\Delta P = f(\rho, \mu, U, L, h) \rightarrow$ D.I: ~~ρ~~, U, h

$$\frac{\Delta P \cdot h}{\mu U} = f\left(\frac{\rho U h}{\mu}, \frac{h}{L}\right) \qquad Re = \frac{\rho U h}{\mu}$$

$Re \to 0$: $\frac{\Delta P h}{\mu U} = f_1\left(\frac{L}{h}\right)$

$\frac{L}{h} \to \infty$: $\frac{\Delta P h}{\mu U} = C \frac{L}{h} \Rightarrow \Delta P = C \cdot \frac{\mu U}{h} \cdot \frac{L}{h}$

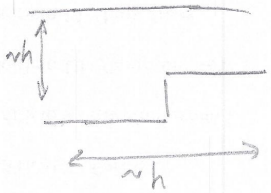
FLOW DOMINATED BY VISCOSITY IF:

$\frac{h}{L} \ll 1$

$\frac{\rho \vec{v} \cdot \nabla \vec{v}}{\mu \nabla^2 \vec{v}} \ll 1 \rightarrow \frac{\rho U^2}{L} \frac{h^2}{\mu U} = \frac{\rho U h}{\mu} \cdot \frac{h}{L} \ll 1$

ESTIMATIONS GIVE: $\Delta P_1 \sim \Delta P_2 \sim \frac{\mu U}{h^2} L$

ADAPTATION REGION



NON SLENDER: $\rho \vec{v} \cdot \nabla \vec{v} = -\nabla P + \mu \nabla^2 \vec{v}$

+

BOUNDARY CONDITIONS.

$\frac{\rho U^2}{h} \qquad \frac{\Delta_E P}{h} \qquad \frac{\mu U}{h^2}$

SINCE $\frac{\frac{\rho U^2}{h}}{\frac{\mu U}{h^2}} = \frac{\rho U h}{\mu} \ll 1. \Rightarrow \Delta_E P \sim \mu \frac{U}{h}$

$\frac{\Delta_E P}{\Delta P_1} = \frac{h}{L} \ll 1.$