ESCUELA POLITÉCNICA SUPERIOR

INTRODUCTION TO FLUID MECHANICS. **14-11-2016**
PROBLEM 1 (10 POINTS). **Time: 50 minutes**

NAME:..... **GROUP:.....**

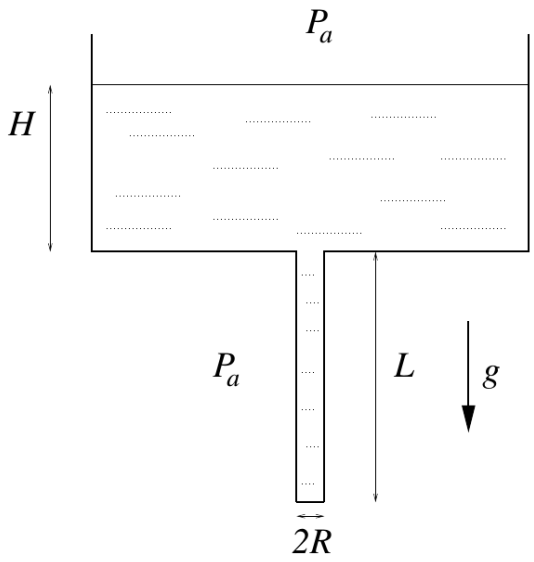
Problema 1 Consider a pipe of length L and radius R, such that $L \gg R$, connected to a deposit of volume $V \simeq L^3$ filled up to a height $H \sim L$ with a liquid of density ρ and viscosity μ, as indicated in the figure. At $t = 0$, the pipe is opened and the liquid comes out of the pipe as a consequence of the gravity force. To describe the discharge process

1. Give the criteria that needs to be satisfied for the viscosity to be dominant in the movement of the fluid inside the pipe. Give, in particular, the entry length in the pipe where the convective terms are comparable to the viscous terms. Also determine the time during which the unsteady terms are important in the discharge process.

2. Obtain the velocity profile $v_z(r, t)$ and the pressure gradient inside the pipe.

3. Determine the evolution of the liquid height $H(t)$ inside the deposit with time.

4. Calculate the discharge time.



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When a micro-organism moves inside a fluid with density ρ and viscosity μ, the Reynolds is usually very small and the problem is dominated by viscosity. If the velocity of micro-organism is $\vec{u} = U_\infty \vec{e}_x$ and its characteristic size is R, we want to determine the drag force on the micro-organism. To do so,

1. Using the Buckingham pi theorem, generate an expression for the drag force D as a function of the other parameters in the problem.

2. Write the equations of Navier-Stokes in non-dimensional form using the characteristics scales giving above. Simplify the system of equations to solve in the limit $Re \to 0$.

3. Using the results of the previous items of the problem, obtain the drag force D as a funtion of an unknown constant Λ.

4. If we assume that the shape of the micro-organism is a sphere of radius R, the velocity and the pressure field are given, in spherical coordinates, by

$$v_r = U_\infty \cos\theta \left[1 - \frac{3R}{2r} + \frac{R^3}{2r^3}\right] \tag{1}$$

$$v_\theta = -U_\infty \sin\theta \left[1 - \frac{3R}{2r} + \frac{R^3}{2r^3}\right] \tag{2}$$

$$p = p_\infty - \frac{3\mu}{2R} U_\infty \cos\theta \tag{3}$$

being p_∞ the pressure far from the surface of the sphere. Using these profiles, calculate the drag force on the sphere

$$D = \int_\Sigma \left[-(p - p_\infty)\cos\theta + \tau_{rr}\cos\theta - \tau_{r\theta}\sin\theta\right] d\sigma \tag{4}$$

and give, in particular, the value of the constant Λ.