INTRODUCTION TO FLUID MECHANICS — PREFINAL (Dic 16, 2013)

A nuclear missile is designed to fly at supersonic speeds (velocities U larger than the sound velocity c) at high altitudes. In the solution that appears, a bow shock is present at a standoff distance d from the missile tip. To characterize the solution and determine the drag force F acting on the missile, a series of experiments is conducted in a supersonic wind tunnel using a small experimental model geometrically similar to the real prototype. Analyze the results following these steps:

1. Use the Π theorem to reduce the parametric dependence of the drag F and standoff distance d. To set up the problem, note that the compressibility of the gas is characterized by the ambient sound speed c, which should be considered at the start of the analysis, along with the ambient values of the density and viscosity of the gas ρ and μ. Since the density changes across the shock, gravity might also influence the solution through the resulting buoyant force.

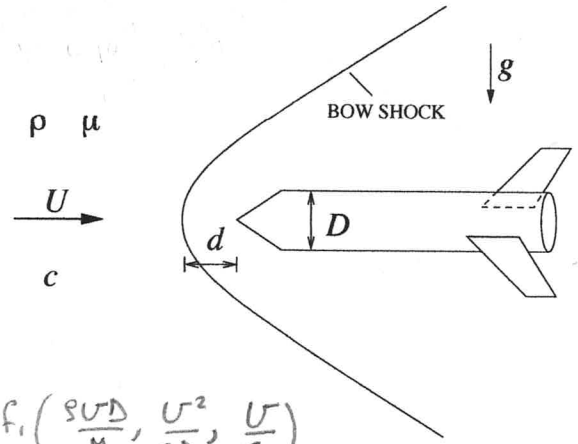
2. Derive the conditions under which both gravity and viscosity do not influence the solution. The reasoning should be substantiated with an order-of-magnitude analysis of the momentum equation to show clearly which dimensionless parameters measure the relative importance of buoyant and viscous forces (relative to the convective acceleration).

3. The experimental tests are carried out at normal ambient conditions ($\rho_e = 1.22$ kg/m^3, $\mu_e = 1.98 \times 10^{-5}$ kg/(m s), $c_e = 340$ m/s) with a model of diameter $D_e = 0.01$ m, significantly smaller than that of the real missile ($D_m = 1$ m), giving the results shown in the table. Use the results, together with the assumption that g and μ have a negligible influence, to deduce the variation with flight speed U_m of the drag force F_m and standoff distance d_m corresponding to the missile flying at cruise altitude, where $\rho_m = 0.36$ kg/m^3, $\mu_m = 1.44 \times 10^{-5}$ kg/(m s), and $c_m = 295$ m/s.

4. Check that the criteria needed to neglect g and μ are satisfied both in the experiment and for the real missile conditions.

5. Determine a simplified expression for the drag force when the Mach number is very large (i.e., $U/c \gg 1$).

U_e (m/s)	F_e (N)	d_e (m)
400	9.2	0.012
700	37.50	0.01
1,000	86.00	0.005
2,000	383	0
4,000	1532	0



1) $F = F(D, U, \rho, \mu, c, g)$, $d = d(D, U, \rho, \mu, c, g)$ $\Rightarrow \frac{F}{\rho U^2 D^2} = f_1\left(\frac{\rho U D}{\mu}, \frac{U^2}{gD}, \frac{U}{c}\right)$

$\frac{d}{D} = f_2\left(\frac{\rho U D}{\mu}, \frac{U^2}{gD}, \frac{U}{c}\right)$

2) $\frac{O(\rho \vec{v}\cdot\nabla\vec{v})}{O(\nabla\cdot\vec{\tau})} = \frac{\rho U D}{\mu}$, $\frac{O(\rho \vec{v}\cdot\nabla\vec{v})}{O(\rho g)} = \frac{U^2}{gD}$ VISCOSITY NEGLIGIBLE IF $\frac{\rho U D}{\mu} \gg 1$; GRAVITY NEGLIGIBLE IF $\frac{U^2}{gD} \gg 1$

3) PHYSICAL SIMILARITY IF $\frac{U_m}{c_m} = \frac{U_e}{c_e} \Rightarrow \boxed{U_m = \frac{c_m}{c_e} U_e}$ $\rightarrow \frac{F_m}{\rho_m U_m^2 D_m^2} = \frac{F_e}{\rho_e U_e^2 D_e^2} \Rightarrow \boxed{F_m = \frac{\rho_m}{\rho_e}\left(\frac{U_m}{U_e}\right)^2 \left(\frac{D_m}{D_e}\right)^2 F_e}$

$\rightarrow \frac{d_m}{D_m} = \frac{d_e}{D_e} \Rightarrow \boxed{d_m = \frac{D_m}{D_e} d_e}$

$\left(\frac{\rho_e U_e D_e}{\mu_e}\right)_{min} = 2.4\times10^5 \gg 1$, $\left(\frac{\rho_m U_m D_m}{\mu_m}\right)_{min} = 8.67\times10^6 \gg 1$

$\left(\frac{U_e^2}{g D_e}\right)_{min} = 1.6\times10^6 \gg 1$, $\left(\frac{U_m^2}{g D_m}\right)_{min} = 12.3\times10^3 \gg 1$

5) FOR $U/c \gg 1$: $\frac{F}{\rho U^2 D^2} = C \simeq 0.785$ ← AS EVALUATED FOR $\frac{U}{c} \gg 1$

U_m (m/s)	F_m (N)	d_m (m)
347	20,437	1.2
607	83,302	1.0
868	1.91×10^4	0.5
1735	8.5×10^5	0
3471	3.4×10^6	0