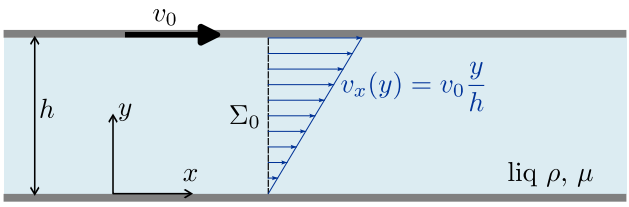
FLUID MECHANICS I **10-11-2025**
SHORT QUESTIONS 1 (10 POINTS) 30%. **Time: 30 minutes**

NAME:..... **GRADE:.....**

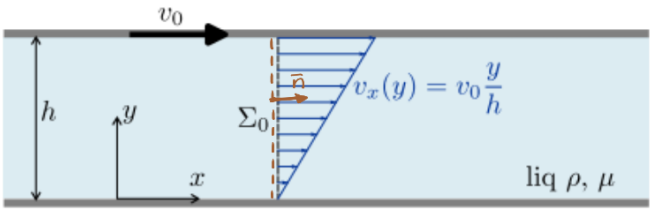
EXERCISE 1: Convective flux

The planar Couette flow between two parallel plates separated a distance h is given by the linear velocity profile shown in the figure.



- Compute the convective fluxes of mass, momentum and kinetic energy across the surface Σ_0 from left to right.
- Explain in words (no equations) what the convective flux of an extensive fluid magnitude is.

The planar Couette flow between two parallel plates separated a distance h is given by the linear profile shown in the figure.



- Compute the convective fluxes of mass, momentum and kinetic energy across the surface Σ_0 from left to right.
- Explain in words (no equations) what the convective flux of an extensive fluid magnitude is.

The following are fluxes p.u. lenght in the 2-direction, across Σ_0 from left to right

$\eta = \frac{y}{h}$

- **Mass flux**

$\bar{n}$ from left to right

$$\int_{y=0}^{y=h} \rho\, \bar{v}\cdot\bar{e}_x\, dy = \int_{y=0}^{y=h} \rho\, v_x(y)\, dy = \int_{y=0}^{y=h} \rho\, v_0 \frac{y}{h}\, dy = \rho v_0 h \int_{\eta=0}^{\eta=1} \eta\, dy = \frac{1}{2}\rho v_0 h$$

$= \rho v_m h \hookrightarrow v_m = \frac{v_0}{2}$ mean velocity

- **Momentum flux**:

$$\int_{y=0}^{y=h} \rho \bar{v}\,(\bar{v}\cdot\bar{e}_x)\, dy = \int_{y=0}^{y=h} \rho\,[v_x(y)]^2\, \bar{e}_x\, dy =$$

$$= \int_{y=0}^{y=h} \rho\, v_0^2 \left(\frac{y}{h}\right)^2 \bar{e}_x\, dy = \rho v_0^2 h \int_{\eta=0}^{\eta=1} \eta^2\, dy\; \bar{e}_x = \frac{1}{3}\rho v_0^2 h\, \bar{e}_x = \frac{4}{3}\rho v_m^2 h \bar{e}_x$$

- **Kinetic energy flux**:

$$\int_{y=0}^{y=h} \frac{1}{2}\rho |\bar{v}|^2 (\bar{v}\cdot\bar{e}_x)\, dy = \int_{y=0}^{y=h} \frac{1}{2}\rho\,[v_x(y)]^3\, dy =$$

$$= \int_{y=0}^{y=h} \frac{1}{2}\rho\, v_0^3 \left(\frac{y}{h}\right)^3 dy = \frac{1}{2}\rho v_0^3 h \int_{\eta=0}^{\eta=1} \eta^3\, dy = \frac{1}{8}\rho v_0^3 h = \rho v_m^3 h = 2\,\frac{1}{2}\rho v_m^3 h$$

Convective flux of an extensive fluid magnitude across a given surface refers to the amount of fluid magnitude that crosses with the fluid the given surface per unit time.

FLUID MECHANICS I **10-11-2025**
PROBLEM 1 (10 POINTS) 70%. **Time: 60 minutes**

NAME:..... **GRADE:.....**

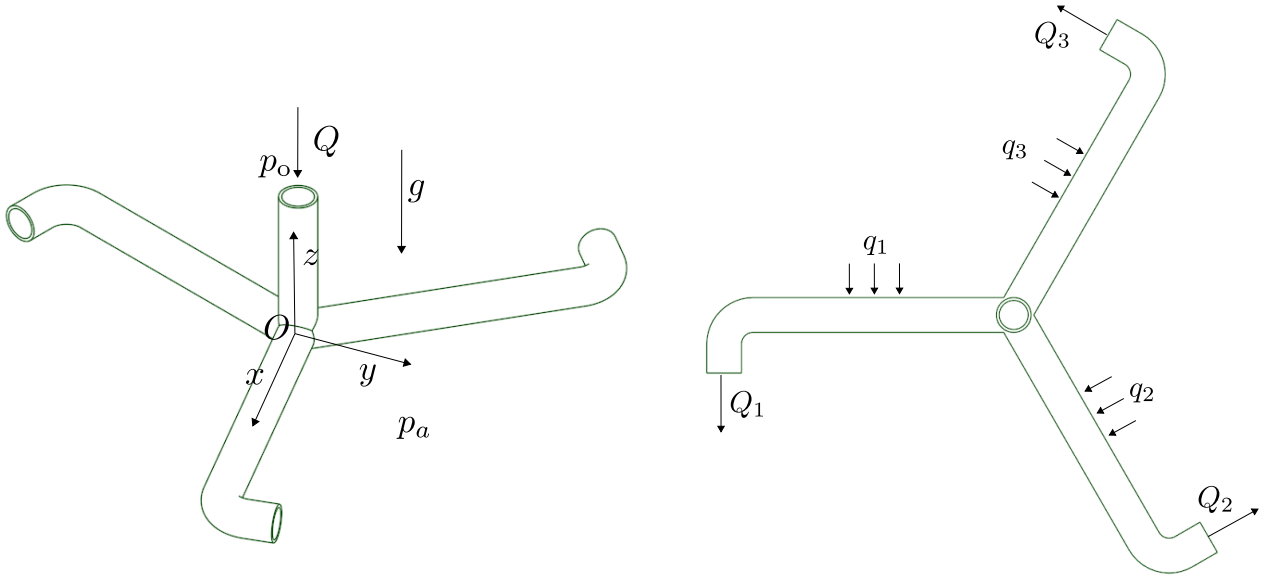
Sprinkler

The figure shows an isometric and a top view of a static sprinkler. A volumetric flow Q of water, of density, viscosity, conductivity and heat capacity ρ, μ, k and c, respectively, enters the sprinkler at a pressure p_o and temperature T_0 through the vertical pipe and is distributed to identical horizontal arms evenly spaced 120º, which discharge to the surrounding atmosphere at pressure p_a. All pipes have a length L and a diameter D. If the total volumetric flow rate is Q,

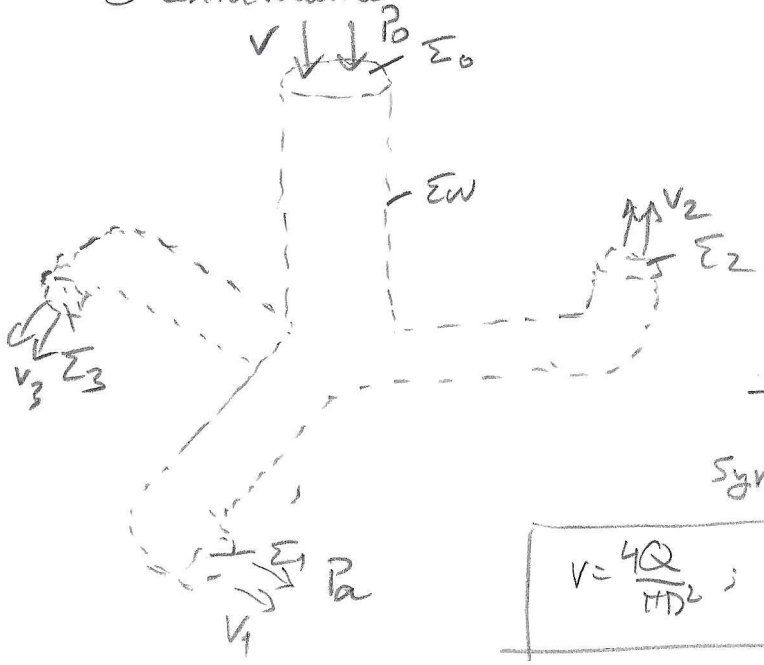
1. Determine the flow rates Q_i and velocities v_i through each of the three branches, with $i = 1, 2, 3$. **(1 Point)**
2. Determine the **net** force $\bar{F}$ exerted by the fluid on the solid structure. Do **not** neglect gravity. **(2.5 Points)**
3. Determine the moment $\bar{M}_O$, about the origin of the reference, exerted on the structure. **(2.5 Points)**

If the temperatures at the exit of the pipes are, respectively $T_1 = T_0$, $T_2 = T_0 + \Delta T$ and $T_3 = T_0 - \Delta T$, the common pipe has adiabatic walls and dissipation is negligible,

4. Determine the heat fluxes q_i added through the walls of each of the pipes. **(3.5 Points)**
5. Use the energy equation to estimate the increment of temperature that the viscous dissipation would produce between the inlet and the origin of the reference system. **(0.5 Points)**



① Control volume



Continuity:

$$\cancel{\frac{d}{dt}\int_{V_c}\rho\,dV} + \int_{\Sigma_c}\rho(\vec{v}-\vec{v}_c)\cdot\vec{n}\,d\sigma = 0$$

Σ_c: Σ_0, Σ_1, Σ_2, $\cancel{\Sigma_3}$, $\cancel{\Sigma_w}$

$$-Q + Q_1 + Q_2 + Q_3 = 0$$

Symmetry $\Rightarrow$ $\boxed{Q_1 = Q_2 = Q_3 = \frac{Q}{3}}$

$$\boxed{V = \frac{4Q}{\pi D^2}\ ;\ V_i = \frac{4Q_i}{\pi D^2} = \frac{4Q}{3\pi D^2}\ ,\ i = 1,2,3}$$

② Momentum in the same C.V.

$$\cancel{\frac{d}{dt}\int_{V_c}\rho\vec{v}\,dV} + \int_{\Sigma_c}\rho\vec{v}(\vec{v}-\vec{v}_c)\cdot\vec{n}\,d\sigma = \int_{\Sigma_c}-P\vec{n}\,d\sigma + \int_{\Sigma_c}\bar{\bar{\tau}}\cdot\vec{n}\,d\sigma + \underbrace{\int_{V_c}\rho\vec{f}_m\,dV}_{-\rho g\,\pi\frac{D^2}{4}\cdot L\cdot 3\,\vec{k}}$$

STEADY; SYMMETRY; Σ_c: Σ_0, Σ_1, Σ_2, $\cancel{\Sigma_3}$, $\cancel{\Sigma_w}$

$$\rho V^2\pi\frac{D^2}{4}\cdot\vec{k} = \int_{\Sigma_c}-P\vec{n}\,d\sigma + \underbrace{\int_{\Sigma_c}P_a\vec{n}\,d\sigma}_{=0} + \int_{\Sigma_w}\bar{\bar{\tau}}\cdot\vec{n}\,d\sigma - \rho g\,\pi\frac{D^2}{4}L\,3\,\vec{k}$$

$$\int_{\Sigma_0}-(P-P_a)\vec{n}\,d\sigma$$

Σ_0; $\cancel{\Sigma_1}$, $\cancel{\Sigma_2}$, $\cancel{\Sigma_3}$, $\cancel{\Sigma_w}$

$$\rho V^2\pi\frac{D^2}{4}\vec{k} = -(P_0-P_a)\,\pi\frac{D^2}{4}\vec{k} - \rho g\,\pi\frac{D^2}{4}L\,4\vec{k} + \underbrace{\int_{\Sigma_w}-(P-P_a)\cdot\vec{n}\,d\sigma + \int_{\Sigma_w}\bar{\bar{\tau}}\cdot\vec{n}\,d\sigma}_{-\vec{F} = -(\vec{F}_{L\to w} + \vec{F}_{A\to w})}$$

$$\boxed{\vec{F} = \left[\rho V^2 + (P_0 - P_a) + 4\rho g L\right]\pi\frac{D^2}{4}(-\vec{k})}$$

(3) ANGULAR MOMENTUM.

$$\frac{d}{dt}\int_{V_c} \bar{x}\wedge \rho\bar{v}\,dV + \int_{\Sigma_c} \bar{x}\wedge \rho\bar{v}\,(\bar{v}-\bar{v}_c)\cdot\bar{n}\,d\sigma = \int_{\Sigma_c} \bar{x}\wedge(-(p-p_a)\bar{n})\,d\sigma + \int_{\Sigma_c} \bar{x}\wedge\bar{\bar{\tau}}\cdot\bar{n}\,d\sigma + \int_{V_c} \bar{x}\wedge\rho\bar{g}\,dV$$

STEADY — SYMMETRY ($\bar{\tau}_0$, $\bar{\tau}_1$, $\bar{\tau}_2$, $\bar{\tau}_3$) — SYMMETRY ($\bar{\tau}_0$, $\bar{\tau}_4$, $\bar{\tau}_5$, $\bar{\tau}_6$) — SYMMETRIC

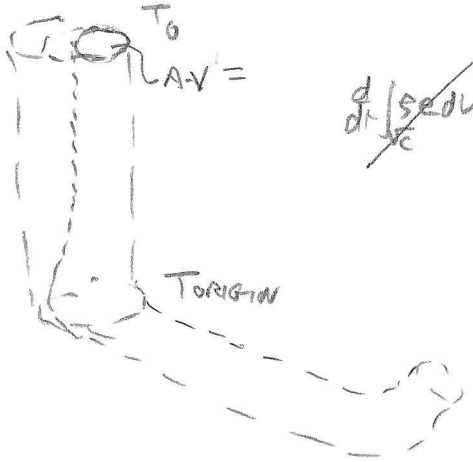
$\bar{\tau}_W$ $\bar{\tau}_W$

$$-\bar{M}_0$$

$$L\rho V_1 \cdot V_1 \cdot \frac{\pi D^2}{4}\bar{k} + L\rho V_2^2\frac{\pi D^2}{4}\bar{k} + L\rho V_3^2\frac{\pi D^2}{4}\bar{k} = -\bar{M}_0$$

$$\boxed{\bar{M}_0 = -\rho V_1^2 \frac{\pi D^2}{4} 3L\,\bar{k}}$$

(4) STREAMTUBE BETWEEN ENTRANCE AND EXIT 1



$$\frac{d}{dt}\int_{V_c} \rho e\,dV + \int_{\Sigma_c} \rho e\,(\bar{v}-\bar{v}_c)\cdot\bar{n}\,d\sigma = \int_{\Sigma_c} -\bar{q}\cdot\bar{n}\,d\sigma + \int \dot{Q}_R + \dot{Q}_{ch}\,dV + \int_{\Sigma_c} -p\,\bar{v}\cdot\bar{n}\,d\sigma + \int_{V_c} \phi_1\,dV$$

Area of stream tube at entrance:

$$A\cdot V = \frac{\pi D^2}{4}\cdot\frac{V}{3} \Rightarrow A = \frac{\pi D^2}{4}\cdot\frac{1}{3}$$

$$-\rho\, V_C\, T_0\, \frac{\pi D^2}{4}\cdot\frac{1}{3} + \rho V_1 \cdot c(T_0)\,\frac{\pi D^2}{4} = q_1$$

$$q_1 = \rho\frac{V}{3}\frac{\pi D^2}{4}\cdot(T_0 - T_0) = 0$$

SIMILARLY IN PIPE 2:

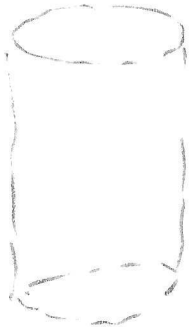
$$-\rho\frac{V_C}{3} T_0 \frac{\pi D^2}{4} + \rho\frac{V}{3}(T_0+\Delta T)\, c\,\frac{\pi D^2}{4} = q_2$$

$$\boxed{q_2 = \rho c \Delta T \frac{V}{3}\frac{\pi D^2}{4}}$$

PIPE 3: $-\rho\frac{V\cdot C}{3} T_0 \frac{\pi D^2}{4} + \rho\frac{V}{3} c\,(T_0-\Delta T)\frac{\pi D^2}{4} = q_3$

$$\boxed{q_3 = -\rho c \Delta T \frac{V}{3}\frac{\pi D^2}{4}}$$

(5) The importance of dissipation along the main pipe can be estimated using the energy eqn.



$$\int_{S_c} \rho c \vec{v}\cdot\vec{n}\,d\sigma = \int_{S_c} -\vec{q}\cdot\vec{n}\,d\sigma + \int_{V_c} (\rho_{ext}+\rho_{ch})\,dV + \int_{V_c} -p\,\nabla\cdot\vec{v}\,dV + \int_{V_c} \phi_v\,dV$$

where $\phi_v \sim \mu\left(\frac{\partial v_i}{\partial r}\right)^2 \sim \mu\left(\frac{V}{D}\right)^2$

$$\rho c\,\Delta T\, V\, D^2 \sim \mu\left(\frac{V}{D}\right)^2 \cdot D^2 \cdot L$$

$$\Delta T \sim \frac{\mu}{\rho V D}\cdot\frac{L}{D}\cdot\frac{V^2}{c}$$

which can be compared with T_0: $\frac{\Delta T}{T_0} \sim \frac{\mu}{\rho V D}\cdot\frac{L}{D}\cdot\frac{V^2}{c\,T_0}$

Typical values: $Re = \frac{\rho V D}{\mu} \sim 10^4$

$$\frac{V^2}{c\,T_0} \sim \frac{10^2}{4\,10^3 \cdot 300}, \quad (V \sim 10\,m/s)$$

$$\frac{\Delta T}{T_0} = \frac{1}{10^8}\cdot\frac{L}{D}, \quad \frac{\Delta T}{T_0} \sim 1 \Rightarrow \frac{L}{D} \sim 10^8\ !!!$$