Midterm20221107.pdf

UC3MEXAMS

Mecánica de Fluidos I

2º Grado en Ingeniería Aeroespacial

Escuela Politécnica Superior. Campus de Leganés
Universidad Carlos III de Madrid

Accede al documento original

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FLUID MECHANICS I **07-11-2022**
PROBLEM 1 (16 POINTS). **Time: 75 minutes**

NAME:..... **GRADE:.....**

The sketch below shows a 2D micro-wind tunnel to test small aerodynamic devices. The length and height are, respectively, $2L$ and $2h$, and the installation discharges in the atmosphere at a pressure p_O. In order to characterize the flow in the installation, first a flow rate Q and temperature T_i is introduced in the empty installation. As L is sufficiently large, consider $\partial u/\partial x = 0$,

1. Use the continuity equation to determine $v(x, y)$.
2. Use the y-momentum equation to show that $p = p(x)$.
3. Use the x-momentum equation to formulate the problem and obtain $u(y)$ as a function of dp/dx, including boundary conditions at the top and bottom walls.
4. Use the last result and the known total flow rate Q to completely determine $p(x)$. In particular, obtain the pressure at the inlet $p_i = p(x = -L)$.
5. Use a suitable control volume to evaluate the force $\bar{F}$ exerted by the air on the walls.

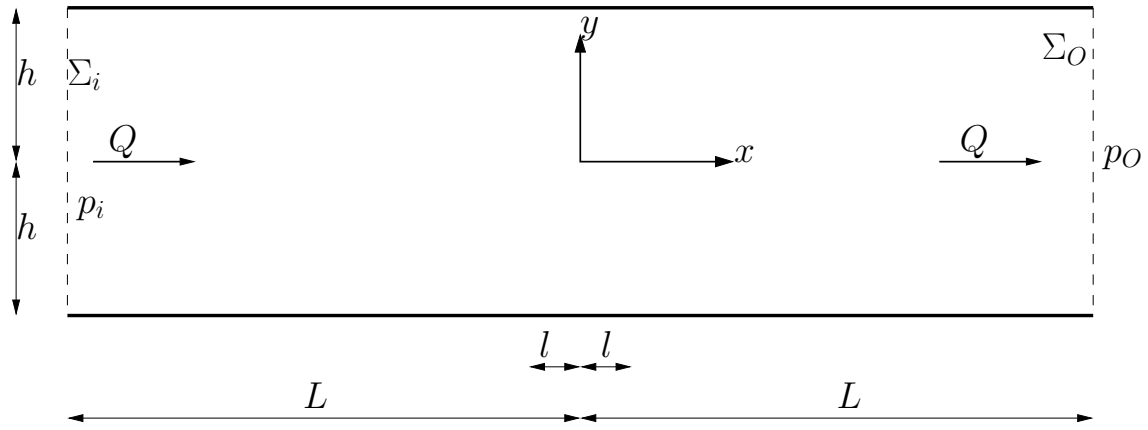
After the empty installation is characterized, an airfoil with film-cooling is placed in the wind-tunnel, as shown in sketch 2. In this airfoil, cool air is injected, normal to the free-stream, through the airfoil. The airfoil, of length $2l << 2L$, is placed in the center of the test cell, as shown in the sketch 2. The airfoil can be assumed to be infinitely thin (flat plate) and uniform flow rates $Q_1 = 2v_1 l$ and $Q_2 = 2v_2 l$ are injected perpendicularly to the free stream through its upper and lower surfaces, with a temperature T_b, to obtain an overal increment of flow rate $\Delta Q = Q_1 + Q_2$. Some assumptions will be made in the analysis. Because of the small size $2l$ of the airfoil, the perturbation can be considered as local and small. Both upstream and downstream of the airfoil the analysis previously carried out remains valid (notice the respective flow rates are Q and $Q + \Delta Q$). In particular, the pressure can be taken as linear in x for $-L < x < 0$ and for $0 < x < L$, and continuous at $x = 0$. If the force exerted on the airfoil is experimentally obtained to be $\bar{F}_b$,

6. Determine the new pressure profiles in presence of the airfoil. In particular, obtain the new values for $p(x = 0)$ and p_i
7. Find the force exerted by the air on the walls
8. Take the walls as adiabatic and prove, using the differential equation, that the temperature at the exit will be uniform if the channel is long enough to ensure $\partial T/\partial x$ becomes 0 before reaching the outlet. Determine the temperature of the gas at the outlet section.

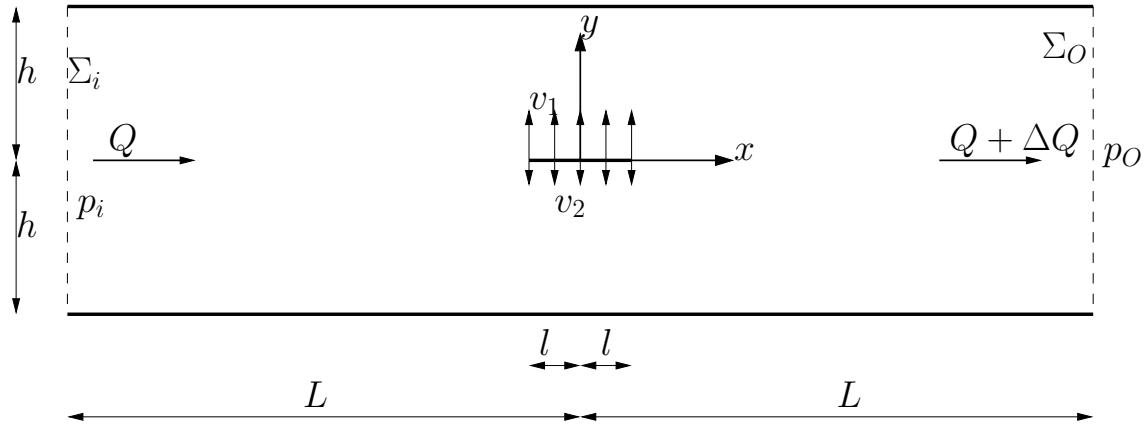
Note: Gravity and Rayleigh's viscous dissipation ϕ_v can be neglected

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Sketch 1

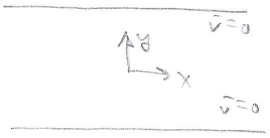


Sketch 2

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① $\frac{\partial v}{\partial y}=0 \Rightarrow \boxed{v=0}$

② $0=-\frac{\partial P}{\partial y} \Rightarrow \boxed{P=P(x)}$



③ $0=-\frac{\partial P}{\partial x}(x)+\mu\frac{\partial^2 u}{\partial y^2}$

$y=h: u=0$

$y=h: u=0$

$\frac{\partial u}{\partial y}=\frac{1}{\mu}\frac{\partial P}{\partial x}y+\cancel{\frac{\partial}{\partial y}}^{0}$ SYMMETRY

$u=\frac{1}{\mu}\frac{\partial P}{\partial x}\frac{y^2}{2}+\beta$

$0=\frac{1}{\mu}\frac{\partial P}{\partial x}\frac{h^2}{2}+\beta \Rightarrow \boxed{u=-\frac{1}{2\mu}\frac{\partial P}{\partial x}(h^2-y^2)}$

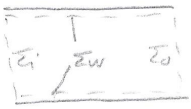
④ $Q=\int_{-h}^{h} u(y)\,dy=-\frac{1}{2\mu}\frac{\partial P}{\partial x}\left(h^2y-\frac{y^3}{3}\right)_{-h}^{h}=-\frac{2h^3}{3\mu}\frac{\partial P}{\partial x} \rightarrow \frac{\partial P}{\partial x}=-\frac{3\mu}{2h^3}Q$

$P=-\frac{3\mu}{2h^3}Qx+C \qquad P(x=L)=P_0$

$\boxed{P=P_0+\frac{3\mu Q}{2h^3}(L-x)} \rightarrow \boxed{P_i=P_0+\frac{3\mu QL}{h^3}}$

$\boxed{u=\frac{3Q}{4h^3}(h^2-y^2)}$

⑤



$\cancel{\frac{d}{dt}\int_{V_c}\rho\vec{v}\,dV}+\int_{\Sigma_c}\rho\vec{v}(\vec{v}-\cancel{\vec{v}_c}^{0})\cdot\vec{n}\,d\sigma=\int_{\Sigma_c}-p\vec{n}\,d\sigma+\int_{\Sigma_c}\vec{\tau}'\cdot\vec{n}\,d\sigma+\cancel{\int_{V_c}\rho\vec{f}_m\,dV}$

$\Sigma_c\begin{cases}\Sigma_i\\ \Sigma_o\\ \Sigma_L\end{cases}$ EQUAL $\quad \vec{v}=\vec{v}_c=0$

$\vec{\tau}_i'\cdot\vec{n}\big|_i=\vec{\tau}_o'\cdot\vec{n}\big|_o \qquad \vec{\tau}'\neq 0\,!!$

$\Sigma_c\begin{cases}\Sigma_i\\ \Sigma_o\\ \Sigma_w\end{cases} \quad \Sigma_w=-\vec{F}$

NO FORCE FROM ATMOSPHERE.

$0=(-P_i\cdot 2h+P_o\,2h)\vec{e}_x-\vec{F}$

$\boxed{\vec{F}=(P_i-P_o)2h\vec{e}_x=\frac{6\mu QL}{h^2}\vec{e}_x}$

⑥ DOWNSTREAM: $0<x<L$: $Q+\Delta Q \rightarrow \frac{\partial P}{\partial x}=-\frac{3\mu}{2h^3}(Q+\Delta Q) \rightarrow \boxed{P=P_0+\frac{3}{2}\frac{\mu}{h^3}(Q+\Delta Q)(L-x)}$

$\boxed{u=\frac{3(Q+\Delta Q)}{4h^3}(h^2-y^2)}$ $\qquad \boxed{P(x=0)=P_0+\frac{3}{2}\frac{\mu}{h^3}(Q+\Delta Q)L}$

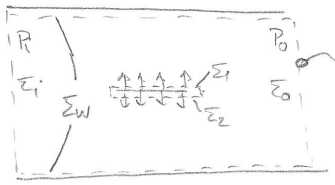
UPSTREAM $-L<x<0$: $Q \rightarrow \frac{\partial P}{\partial x}=-\frac{3\mu}{2h^3}Q \rightarrow \boxed{P=P(x=0)-\frac{3\mu}{2h^3}Qx}$

$\boxed{u=\frac{3Q}{4h^3}(h^2-y^2)}$ $\qquad \boxed{P_i=P(x=-L)=P_0+\frac{3\mu}{2h^3}(Q+\Delta Q)L+\frac{3\mu}{2h^3}QL}$

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(7)



$$\bar{\bar{\tau}}' = \mu \begin{pmatrix} 0 & -\frac{3(Q+\Delta Q)}{4h^3} 2y \\ -\frac{3(Q+\Delta Q)}{4h^3} 2y & 0 \end{pmatrix}$$

$$\bar{\bar{\tau}}' \cdot \bar{n}\Big|_{\Sigma_0} = -\mu \frac{\partial u}{\partial y}\Big|_{(-L,y)} \bar{e}_y \rightarrow \int_{\Sigma_0} \bar{\bar{\tau}}' \cdot \bar{n}\, d\sigma = -\mu \int_{-h}^{h} \frac{\partial u}{\partial y} dy = 0$$

SAME THING AT Σ_i

$$\frac{d}{dt}\int_V \rho \bar{v}\, dV + \int_{\Sigma_c} \rho \bar{v}(\bar{v} - \bar{v}_c)\cdot \bar{n}\, d\sigma = \int_{\Sigma_c} -p\bar{n}\, d\sigma + \int_{\Sigma_c} \bar{\bar{\tau}}' \cdot \bar{n}\, d\sigma + \int_{V_c} \rho \bar{f}\, dV$$

Σ_c: Σ_i, Σ_0, ~~Σ_W~~, Σ_1, Σ_2 — $\bar{\bar{\tau}}'$: ~~Σ_i~~, ~~Σ_0~~, Σ_W, Σ_1, Σ_2

Σ_W: $= -\bar{F}$ — Σ_1, Σ_2: $= -\bar{F}_b$

$$\int_{\Sigma_i} \rho \bar{v}\,\bar{v}\cdot\bar{n}\, d\sigma = -\rho \int_{-h}^{h} \left(\frac{3Q}{4h^3}\right)^2 (h^2 - y^2)\, dy\, \bar{e}_x = -\rho \left(\frac{3Q}{4h^3}\right)^2 \left[\frac{16}{15} h^5\right] \bar{e}_x$$

$$\int_{\Sigma_0} \rho \bar{v}\,\bar{v}\cdot\bar{n}\, d\sigma = \rho \left(\frac{3(Q+\Delta Q)}{4h^3}\right)^2 \frac{16}{15} h^5\, \bar{e}_x$$

$$\int_{\Sigma_1} \rho \bar{v}\,\bar{v}\cdot\bar{n}\, d\sigma = -\rho v_1^2 2L\, \bar{e}_y$$

$$\int_{\Sigma_2} \rho \bar{v}\,\bar{v}\cdot\bar{n}\, d\sigma = \rho v_2^2 2L\, \bar{e}_y$$

$$\rho \frac{9}{16} \frac{1}{h} \frac{16}{15} \left[-Q^2 + Q^2 + 2Q\Delta Q + \Delta Q^2\right] - \rho \frac{Q_1^2}{2L} \bar{e}_y + \rho \frac{Q_2^2}{2L} \bar{e}_y =$$

$$= (P_i - P_0) 2h\, \bar{e}_x - \bar{F} - \bar{F}_b$$

$$\boxed{\bar{F} = \left\{ \frac{3\mu QL}{h^3} + \frac{3}{2}\frac{\mu \Delta Q L}{h^3} - \frac{9}{15}\frac{\rho \Delta Q}{h}(2Q + \Delta Q) \right\} \bar{e}_x + \frac{\rho (Q_1^2 - Q_2^2)}{2L} \bar{e}_y - \bar{F}_b}$$

(8) PDE ENERGY

$$\rho \frac{\partial e}{\partial t} + \rho \bar{v}\nabla e = \nabla \cdot (k \nabla T) \qquad \frac{\partial T}{\partial x} = 0$$

$$\left.\begin{array}{l} \frac{\partial T}{\partial y} = 0 \\ y = -h: \frac{\partial T}{\partial y} = 0 \\ y = h: \frac{\partial T}{\partial y} = 0 \end{array}\right\} \rightarrow \frac{\partial T}{\partial y} = \text{const} = 0$$

$T = T_0$, TO BE DETERMINED

$$\frac{d}{dt}\int_V \rho c T\, dV + \int_{\Sigma_c} \rho c T \bar{v}\cdot\bar{n}\, d\sigma = \int_{\Sigma_c} -\bar{q}\cdot\bar{n}\, d\sigma$$

$$\rho c T_i \underbrace{\int_{\Sigma_i} \bar{v}\cdot\bar{n}\, d\sigma}_{-Q} + \rho c T_0 \underbrace{\int_{\Sigma_0} \bar{v}\cdot\bar{n}\, d\sigma}_{Q+\Delta Q} + \rho c T_b \underbrace{\int_{\Sigma_1} \bar{v}\cdot\bar{n}\, d\sigma}_{-Q_1} + \rho c T_b \underbrace{\int_{\Sigma_2} \bar{v}\cdot\bar{n}\, d\sigma}_{-Q_2} = 0$$

$$\boxed{T_0 = \frac{Q T_i + \Delta Q T_b}{Q + \Delta Q} = T_i + \frac{\Delta Q (T_b - T_i)}{Q + \Delta Q}}$$

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FLUID MECHANICS I **07-11-2022**
EXERCISE 2 (4 POINTS). **Time: 20 minutes**

NAME:..... **GRADE:.....**

The objective is to derive the PDE that governs the mean temperature field $T_m(x,t)$ (1-dimensional, unsteady) for a liquid with constant properties that flows in a tube of cross section area A and adiabatic walls. The flow is characterized by a uniform, steady velocity field $\bar{v} = U_m \bar{e}_x$.

1. Apply the internal energy equation to the cylindrical C.V. defined in the figure, with length Δx and cross section area A.
2. Divide the equation by the volume of the C.V. and take the limit $\Delta x \to 0$.

Note 1: Internal energy equation:

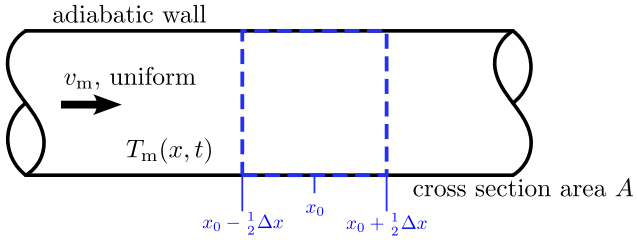
$$\frac{d}{dt}\int_{V_c} \rho e dV + \int_{\Sigma_c} \rho e(\bar{v}-\bar{v}_c)\cdot\bar{n}d\sigma = -\int_{\Sigma_c}\bar{q}\cdot\bar{n}d\sigma + \int_{V_c}(Q_{ch}+Q_R)dV + \left[-\int_{V_c} p\nabla\cdot\bar{v}dV + \int_{V_c}\phi_v dV\right]$$

with $e = cT$ and $\bar{q} = -k\nabla T$

Note 2: Viscous dissipation, radiation and chemical reaction effects can be neglected

Note 3: $T_m(x,t)$ is sufficiently smooth and use can be made of Taylor series, near a point x_0, according to

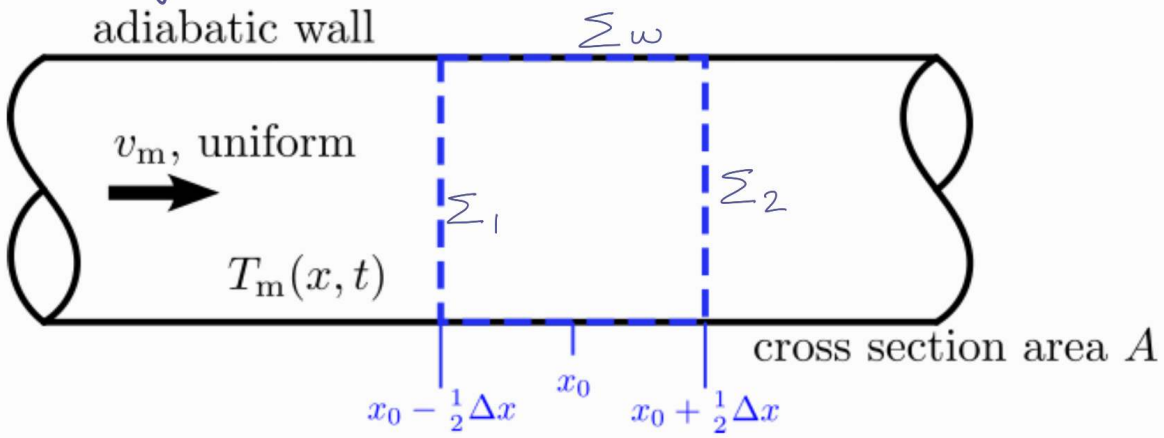
$$T_m(x,t) = T_m(x_0,t) + (x-x_0)\left.\frac{\partial T_m}{\partial x}\right|_{(x_0,t)} + \frac{(x-x_0)^2}{2}\left.\frac{\partial^2 T_m}{\partial x^2}\right|_{(x_0,t)} + \dots$$



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for a generic x_0



$$\frac{d}{dt}\int_{V_c} \rho c T_m \, dV + \int_{\Sigma_c} \rho c T_m \, \bar{v}\cdot\bar{n} \, d\Sigma = + \int_{\Sigma_c} \kappa \, \nabla T_m \cdot \bar{n} \, d\Sigma$$

$$\int_{x_0-\frac{\Delta x}{2}}^{x_0+\frac{\Delta x}{2}} \rho c \, T_m(x,t) \, A \, dx$$

Σ_1 $\quad -\rho c T_m(x_0-\frac{\Delta x}{2},t)\, v_m A$ $\qquad$ Σ_1 $\quad -\kappa \left.\frac{\partial T_m}{\partial x}\right|_{(x_0-\frac{\Delta x}{2},t)} A$

Σ_2 $\quad \rho c T_m(x_0+\frac{\Delta x}{2},t)\, v_m A$ $\qquad$ Σ_2 $\quad \kappa \left.\frac{\partial T_m}{\partial x}\right|_{(x_0+\frac{\Delta x}{2},t)} A$

Σ_w $\quad (\bar{v}\cdot\bar{n}) = 0$ $\qquad$ Σ_w (adiabatic)

Use Taylor series to express all appearances of T and its derivatives evaluated at (x_0,t)

$$\frac{d}{dt}\int_{x_0-\frac{\Delta x}{2}}^{x_0+\frac{\Delta x}{2}} \rho c \left[T_m(x_0,t) + (x-x_0)\left.\frac{\partial T_m}{\partial x}\right|_{(x_0,t)} + \frac{(x-x_0)^2}{2}\left.\frac{\partial^2 T_m}{\partial x^2}\right|_{(x_0,t)} + \cdots \right] A\,dx$$

$$- \rho c \left[T_m(x_0,t) - \frac{\Delta x}{2}\left.\frac{\partial T_m}{\partial x}\right|_{(\,)} + \frac{\Delta x^2}{8}\left.\frac{\partial^2 T_m}{\partial x^2}\right|_{(\,)} + O(\Delta x^3) \right] v_m A$$

$$+ \rho c \left[T_m(x_0,t) + \frac{\Delta x}{2}\left.\frac{\partial T_m}{\partial x}\right|_{(\,)} + \frac{\Delta x^2}{8}\left.\frac{\partial^2 T_m}{\partial x^2}\right|_{(\,)} + O(\Delta x^3) \right] v_m A =$$

$$- \kappa \left[\left.\frac{\partial T_m}{\partial x}\right|_{(x_0,t)} - \frac{\Delta x}{2}\left.\frac{\partial^2 T_m}{\partial x^2}\right|_{(\,)} + \frac{\Delta x^2}{8}\left.\frac{\partial^3 T_m}{\partial x^3}\right|_{(\,)} + O(\Delta x^3) \right] A$$

$$+ \kappa \left[\left.\frac{\partial T_m}{\partial x}\right|_{(\,)} + \frac{\Delta x}{2}\left.\frac{\partial^2 T_m}{\partial x^2}\right|_{(\,)} + \frac{\Delta x^2}{8}\left.\frac{\partial^3 T_m}{\partial x^3}\right|_{(\,)} + O(\Delta x^3) \right] A$$

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Evaluating the volume integral

$$\int_{x_0-\frac{\Delta x}{2}}^{x_0+\frac{\Delta x}{2}} \rho c \left[T_m(x_0,t) + (x-x_0)\left.\frac{\partial T_m}{\partial x}\right|_{(x_0,t)} + \frac{(x-x_0)^2}{2}\left.\frac{\partial^2 T_m}{\partial x^2}\right|_{(x_0,t)} + \cdots \right] A\,dx =$$

$$= \rho c \int_{(x-x_0)=-\frac{\Delta x}{2}}^{\frac{\Delta x}{2}} \left[T_m(x_0,t) + (x-x_0)\left.\frac{\partial T_m}{\partial x}\right|_{(x_0,t)} + \frac{(x-x_0)^2}{2}\left.\frac{\partial^2 T_m}{\partial x^2}\right|_{(x_0,t)} + \cdots \right] A\,d(x-x_0)$$

$$= \rho c A \left[T_m(x_0,t)\,(x-x_0) + \left.\frac{\partial T_m}{\partial x}\right|_{(\,)} \frac{(x-x_0)^2}{2} + \left.\frac{\partial^2 T_m}{\partial x^2}\right|_{(\,)} \frac{(x-x_0)^3}{6} + \cdots \right]_{x-x_0=-\frac{\Delta x}{2}}^{x-x_0=\frac{\Delta x}{2}}$$

$$= \rho c A \left[T_m(x_0,t)\,\Delta x + \left.\frac{\partial^2 T_m}{\partial x^2}\right|_{(\,)} \frac{\Delta x^3}{24} + \cdots \right]$$

Dividing by $A\Delta x$ and taking the limit when $\Delta x \to 0$

$$\rho c \frac{\partial T_m}{\partial t} + \rho c v_m \frac{\partial T_m}{\partial x} = \kappa \frac{\partial^2 T_m}{\partial x^2}$$

$$\alpha \equiv \frac{\kappa}{\rho c}$$

$$\boxed{\frac{\partial T_m}{\partial t} + v_m \frac{\partial T_m}{\partial x} = \alpha \frac{\partial^2 T_m}{\partial x^2}}$$

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