INTRODUCTION TO FLUID MECHANICS — QUIZ (Sep 29, 2014)

Given the steady planar motion described in cylindrical coordinates by the velocity field

$$v_r = Ar\cos(2\theta) \qquad \text{and} \qquad v_\theta = -Ar\sin(2\theta),$$

where A is a positive constant,

1. Calculate the stagnation points. $r=0$
2. Determine the value of the volume dilation rate $\nabla \cdot \bar{v} = 0$
3. Obtain the vorticity $\nabla \wedge \bar{v} = \left[\frac{1}{r}\left(\frac{\partial (r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}\right)\right]\bar{e}_z = 0$
4. Compute the stream line that intersects the point $(r,\theta) = (r_o, \theta_o)$. Sketch the resulting streamlines, including those crossing the origin. $r^2 \sin(2\theta) = r_o^2 \sin(2\theta_o)$
5. Calculate the trajectory of a fluid particle located initially at $(r,\theta) = (r_o, \theta_o)$. $r^2\sin(2\theta) = r_0^2 \sin(2\theta_0)$, $\tan\theta = \tan\theta_0\, e^{-2At}$
6. Give the equation for the fluid line initially defined by $\theta_o = \pi/4$ with $0 \le r < \infty$. $r_0 = \lambda$, $\theta_0 = \pi/4$ → $r^2\sin(2\theta) = \lambda^2$, $\tan\theta = e^{-2At}$ $\Rightarrow \theta = \tan^{-1}\left(e^{-2At}\right)$
7. If it exists, calculate the velocity potential φ. $\varphi = A\frac{r^2}{2}\cos(2\theta) + \varphi_0$
8. If it exists, calculate the stream function ψ, verifying that the streamlines correspond to the isolines ψ =constant. $\psi - \psi_0 = A\frac{r^2}{2}\sin(2\theta)$
9. Determine the acceleration (a_r, a_θ). $\bar{a} = \frac{\partial \bar{v}}{\partial t} + \nabla\left(\frac{v^2}{2}\right) - \bar{v} \wedge (\nabla \wedge \bar{v}) = A^2 r\, \bar{e}_r$, $\frac{v^2}{2} = \frac{A^2 r^2}{2}$ → $a_r = A^2 r$, $a_\theta = 0$
10. Obtain the circulation along the line $\theta = 3\pi/4$. $\int \bar{v}\cdot d\bar{\ell} = \int_0^\infty v_r\left(r, \theta = \frac{3\pi}{4}\right) dr = 0$, with $d\bar{\ell} = dr\,\bar{e}_r$ and $v_\theta = 0$