INTRODUCTION TO FLUID MECHANICS — QUIZ (Sep 30, 2013)

Given the steady planar motion described by the velocity components

$$v_x = Ae^{\Omega t}y \qquad \text{and} \qquad v_y = Be^{\Omega t}x,$$

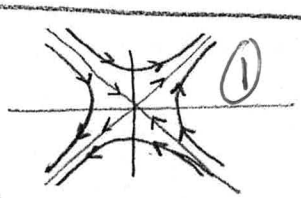
where A, B, and Ω are constants.

1. Determine the value of the volume dilation rate $\nabla \cdot \bar{v}$.
2. Obtain the value of B for which the resulting flow is irrotational. Adopt this particular value in what follows.
3. Calculate the stream line that intersects the point $(x, y) = (x_o, y_o)$ at a given instant of time t. Sketch the resulting stream lines.
4. Compute the trajectory of a fluid particle located initially at $(x, y) = (x_o, y_o)$.
5. If they exist, calculate the velocity potential φ and the stream function ψ.
6. Determine the rate-of-strain tensor $\bar{\bar{T}}_d$.
7. Describe the deformation of a fluid element of square shape whose sides of length dl are initially oriented parallel to the axes.

1) $\nabla \cdot \bar{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$ (1)

2) $\nabla \wedge \bar{v} = \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\bar{e}_z = Be^{\Omega t} - Ae^{\Omega t} = 0 \Rightarrow \boxed{B = A}$ (1)

3) $\frac{dx}{v_x} = \frac{dy}{v_y} \Rightarrow \frac{dx}{y} = \frac{dy}{x} \rightarrow \boxed{y^2 - x^2 = y_o^2 - x_o^2}$ (1)


(1)

4) $\frac{dx}{dt} = Ae^{\Omega t}y$, $\frac{dy}{dt} = Ae^{\Omega t}x$ $\Big\}$ $t=0$: $x=x_o$, $y=y_o$

$\frac{dx}{dy} = \frac{y}{x} \rightarrow \boxed{y^2 - x^2 = y_o^2 - x_o^2}$ (1) PATH LINES $\equiv$ STREAM LINES

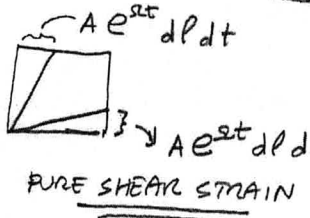
$y = \pm\sqrt{x^2 + y_o^2 - x_o^2}$

$$\int_{x_o}^{x} \frac{dx}{\sqrt{x^2 + y_o^2 - x_o^2}} = \pm A\int_0^t e^{\Omega t}\,dt \Rightarrow \ln\left(\frac{x + \sqrt{x^2 + y_o^2 - x_o^2}}{x_o + y_o}\right) = \pm\frac{A}{\Omega}\left(e^{\Omega t} - 1\right)$$ (1)

5) $v_x = \frac{\partial \varphi}{\partial x} = Ae^{\Omega t}y$, $v_y = \frac{\partial \varphi}{\partial y} = Ae^{\Omega t}x \Rightarrow \boxed{\varphi = Ae^{\Omega t}xy + \varphi_o}$ (1)

$v_x = \frac{\partial \psi}{\partial y} = Ae^{\Omega t}y$, $-v_y = \frac{\partial \psi}{\partial x} = -Ae^{\Omega t}x$ $\Big\}$ $\boxed{\psi = Ae^{\Omega t}(y^2 - x^2) + \psi_o}$ (1)

6) $\bar{\bar{T}}_d = \frac{1}{2}\left(\nabla\bar{v} + \nabla\bar{v}^T\right) = \begin{bmatrix} 0 & Ae^{\Omega t} \\ Ae^{\Omega t} & 0 \end{bmatrix}$ (1)

7) 

PURE SHEAR STRAIN (1)