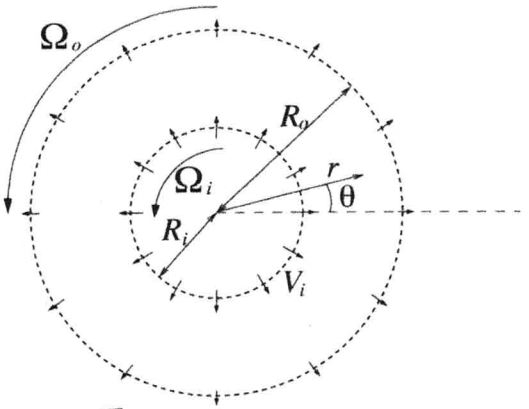
INTRODUCTION TO FLUID MECHANICS **MIDTERM (Nov 11, 2013)**

A liquid of density ρ, viscosity μ, thermal conductivity k, and specific heat c occupies the gap formed between two porous coaxial cylinders of radius R_i and R_o that are rotating with angular velocities Ω_i and Ω_o, respectively. The liquid is injected through the surface of the inner cylinder with normal velocity component $v_r = V_i$ and is removed through the outer surface. The temperatures on the inner and outer surfaces are T_i and T_o, respectively. The resulting steady, planar, axially symmetric flow field is described in terms of $v_r(r)$, $v_\theta(r)$, $p(r)$, and $T(r)$.

1. Determine the radial velocity component $v_r(r)$. Obtain in particular its value on the surface of the outer cylinder $v_r(R_o)$.
2. Obtain the azimuthal velocity component $v_\theta(r)$. To that end, write the θ-component of the momentum equation in terms of the parameter $Re = \rho V_i R_i/\mu$ and look for solutions of the form $v_\theta \propto r^\alpha$, demonstrating that the general solution can be expressed as $v_\theta = C_1/r + C_2 r^{Re+1}$. Obtain the values of the integration constants C_1 and C_2.
3. Calculate $p(r) - p_i$, the difference of the pressure $p(r)$ from its inner-surface value $p_i = p(R_i)$.
4. Write the equation that determines the temperature distribution $T(r)$, including the appropriate boundary conditions. Solve the problem assuming that the effect of viscous dissipation is negligibly small.



1) CONTINUITY $\frac{1}{r}\frac{\partial}{\partial r}(rv_r)=0 \rightarrow \boxed{rv_r = R_i V_i} \rightarrow v_r(R_o) = \frac{R_i V_i}{R_o}$ (1)

2) $\rho v_r \frac{\partial v_\theta}{\partial r} + \rho \frac{v_r v_\theta}{r} = \mu \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) \Rightarrow \frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(rv_\theta)\right) - Re\left(\frac{1}{r}\frac{dv_\theta}{dr} + \frac{v_\theta}{r^2}\right)=0$ (1)

$v_\theta \propto r^\alpha,\ (\alpha+1)(\alpha-1-Re)=0$ (1)

$\boxed{v_\theta = C_1/r + C_2 r^{Re+1}}$

$v_\theta(R_i) = \Omega_i R_i = \frac{C_1}{R_i} + C_2 R_i^{Re+1}$

$v_\theta(R_o) = \Omega_o R_o = \frac{C_1}{R_o} + C_2 R_o^{Re+1}$

$\boxed{C_1 = (R_i R_o)^2 \frac{\Omega_i R_o^{Re} - \Omega_o R_i^{Re}}{R_o^{Re+2} - R_i^{Re+2}},\ C_2 = \frac{\Omega_o R_o^2 - \Omega_i R_i^2}{R_o^{Re+2} - R_i^{Re+2}}}$ (1)

3) $\frac{dP}{dr} = -\rho\left(v_r\frac{dv_r}{dr} - \frac{v_\theta^2}{r}\right) = \rho\frac{(R_i V_i)^2 + C_1^2}{r^3} + \rho C_2^2 r^{2Re+1} + \rho 2C_1C_2 r^{Re-1} \rightarrow$ (1)

$\boxed{\frac{P-P_i}{\rho} = \frac{(R_i V_i)^2 + C_1^2}{2}\left(\frac{1}{R_i^2} - \frac{1}{r^2}\right) + \frac{C_2^2}{2Re+2}\left(r^{2Re+2} - R_i^{2Re+2}\right) + \frac{2C_1C_2}{Re}\left(r^{Re} - R_i^{Re}\right)}$ (1)

4) $\rho c \frac{R_i V_i}{r}\frac{dT}{dr} = \Phi_r + \frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)$ (1), $T(R_i)=T_i$, $T(R_o)=T_o$

IF $\Phi_r \approx 0$ $\rightarrow M/(kC)$

$\Rightarrow \boxed{\frac{d}{dr}\left(r\frac{dT}{dr}\right) - Pr\,Re\frac{dT}{dr} = 0}$ (1)

$\Phi_r = \mu\left\{2\left[\left(\frac{dv_r}{dr}\right)^2 + \left(\frac{v_r}{r}\right)^2\right] + \left[r\frac{d}{dr}\left(\frac{v_\theta}{r}\right)\right]^2\right\} = \mu\left[\frac{4\left(V_i^2 R_i^2 + C_1^2\right)}{r^4} + Re^2 C_2^2 r^{2Re} - 4Re\,C_1C_2 r^{Re-2}\right]$

$\boxed{T = B_1 + B_2 r^{Pr\,Re} = T_i + (T_o - T_i)\frac{r^{Pr\,Re} - R_i^{Pr\,Re}}{R_o^{Pr\,Re} - R_i^{Pr\,Re}}}$ (1)

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The ejector shown in the figure is used to discharge a container filled with a perfect liquid of density ρ and constant specific heat c into the atmosphere, also at pressure p_a. A high-velocity jet of the same liquid, with velocity U_j and cross section A_j, is injected along the axis of the discharge pipe, whose cross section is A. The jet entrains fluid as it develops, producing an underpressure at the injection section and drawing the fluid from the container. Mixing of the jet with the surrounding liquid is completed downstream, so that at the outlet the liquid discharges as a uniform jet with velocity U_o. For a given the geometry (i.e., given values of A_j and A), we want to compute the characteristics of the flow in terms of the injection velocity U_j. In the computation, assume that the effect of gravity is negligible. Address in order the following questions:

1. Assuming that the acceleration of the fluid upstream from the injector is steady and inviscid, write down the equation that relates the ambient pressure p_a with the velocity and pressure at the entrance of the pipe, U_e and p_e.

2. Apply the continuity and momentum balance equations to the control volume indicated in the figure. Simplify the result by assuming that the effect of friction on the wall surface is negligibly small.

3. Use the previous equations to show that the volumetric discharge flow rate is given by

$$Q = U_e(A - A_j) = \frac{U_j(A - A_j)}{1 + A/\sqrt{2A_j(A - A_j)}}$$

and that the velocity at the outlet satisfies

$$\frac{U_o}{U_j} = \frac{A_j + \sqrt{2A_j(A - A_j)}}{A + \sqrt{2A_j(A - A_j)}}.$$

4. Assuming that the values of the temperatures T_e and T_j at the entrance section are equal to the ambient value T_a, obtain the temperature T_o at the outlet section when the pipe walls are adiabatic.

