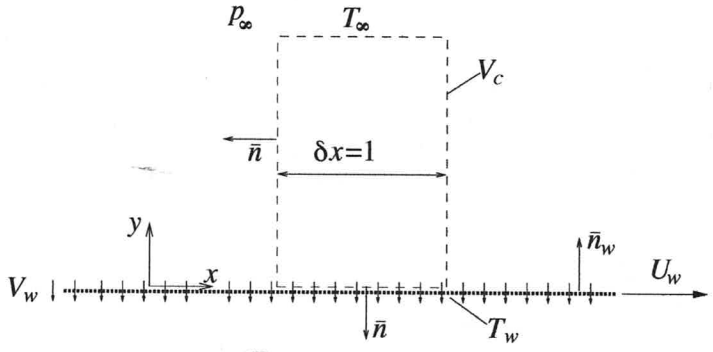
Problem 2: Consider a buoyancy-free, steady planar motion occurring in a semi-infinite space filled with a perfect liquid of density ρ, viscosity μ, thermal conductivity k, and specific heat c. The liquid is bounded by an infinitely long porous wall that moves parallel to itself with velocity U_w while removing the liquid with a normal suction velocity V_w . Far from the wall, the horizontal velocity component is zero and the pressure is $p = p_\infty$. The ambient value of the temperature $T = T_\infty$ is higher than the value T_w at the wall.

1. Determine the velocity components $v_x(y)$ and $v_y(y)$ and well as the pressure field $p(x, y)$.
2. Obtain the temperature distribution $T(y)$.
3. Compute the viscous force acting on the wall per unit surface $\tau_w \bar{e}_x = \bar{\bar{\tau}}' \cdot \bar{n}_w$, where $\bar{n}_w$ is the unit vector normal to the wall.
4. Determine the heat flux from the fluid to the wall $q_w = k(\nabla T) \cdot \bar{n}_w$.
5. For an alternative computation procedure for τ_w and q_w, apply the momentum and energy conservation equations to the control volume indicated in the figure. In dealing with the balance equations, evaluate explicitly the contribution of each term, stating clearly the reasons justifying any simplifications introduced.



1) FROM CONTINUITY $\frac{\partial V_y}{\partial y} = 0 \rightarrow \boxed{V_y = -V_w}$ Y-mom: $0 = -\frac{\partial P}{\partial y} \rightarrow \boxed{P = P_\infty}$

X-mom: $-\rho V_w \frac{dV_x}{dy} = \mu \frac{d^2 V_x}{dy^2}$ $\begin{cases} V_x(0) = U_w \\ V_x(\infty) = 0 \end{cases}$ $\boxed{V_x = U_w e^{-\frac{V_w}{\nu} y}}$

2) $-\rho c V_w \frac{dT}{dy} = \Phi_v + k \frac{d^2 T}{dy^2}$ $\begin{cases} T(0) = T_w \\ T(\infty) = T_\infty \end{cases} \rightarrow \boxed{T = T_\infty + \left(T_w - T_\infty - \frac{Pr}{2(Pr-2)} \frac{U_w^2}{c}\right) e^{-\frac{V_w}{\alpha} y} + \frac{Pr}{2(Pr-2)} \frac{U_w^2}{c} e^{-2\frac{V_w}{\nu} y}}$

$\Phi_v = \mu \left(\frac{dV_x}{dy}\right)^2 = \mu U_w^2 \frac{V_w^2}{\nu^2} e^{-2\frac{V_w}{\nu} y}$ $Pr = \frac{\nu}{\alpha}$, $\alpha = \frac{k}{\rho c}$

3) $\begin{bmatrix} 0 & 0 \\ 0 & \tau_{yy} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \tau_{xy} = \mu \frac{dV_x}{dy}\Big|_{y=0} \bar{e}_x = \boxed{-\rho U_w V_w \bar{e}_x}$

4) $q_w = k \frac{\partial T}{\partial y}\Big|_{y=0} = \boxed{\rho c V_w \left[T_\infty - T_w + \frac{1}{2} \frac{U_w^2}{c}\right]}$

5) $\int_{\Sigma_c} \rho \bar{v} \, \bar{v} \cdot \bar{n} \, d\sigma = -\int_{\Sigma_c} p \bar{n} \, d\sigma + \int_{\Sigma_c} \bar{\bar{\tau}}' \cdot \bar{n} \, d\sigma$

$\rho V_w U_w \bar{e}_x = \bar{\bar{\tau}}' \cdot \bar{n} = -\tau_w \bar{e}_x \rightarrow \boxed{\tau_w = -\rho V_w U_w}$

$\int_{\Sigma_c} \rho \left(e + \frac{v^2}{2}\right) \bar{v} \cdot \bar{n} \, d\sigma = -\int_{\Sigma_c} p \bar{v} \cdot \bar{n} \, d\sigma + \int_{\Sigma_c} \bar{v} \cdot \bar{\bar{\tau}}' \cdot \bar{n} \, d\sigma - \int_{\Sigma_c} \bar{q} \cdot \bar{n} \, d\sigma$

$-\rho V_w \left(c T_\infty + \frac{V_w^2}{2}\right) + \rho V_w \left(c T_w + \frac{U_w^2 + V_w^2}{2}\right) = P_\infty V_w - P_\infty V_w + \rho U_w^2 V_w - q_w$

$\boxed{q_w = \frac{1}{2} \rho V_w U_w^2 + \rho c V_w (T_\infty - T_w)}$