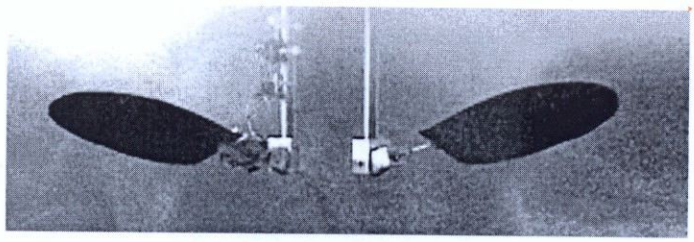
INTRODUCTION TO FLUID MECHANICS

Dec 16, 2011

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A paper published in Scientific American in 2001 by Prof. Michael Dickinson describes an experimental facility designed to determine the lift L generated by a fly suspended at a fixed point by the flapping motion of its wings of length L at a frequency f. The facility includes a large container filled with mineral oil. A pair of wings, geometrically similar to those of the fly, is placed within the container. Programmable electrical engines move the wings, reproducing the wing motion of the fly at varying frequencies. To mimic the characteristic fly dynamics, the wings in the experiment are made out of a very light material, so that their weight and inertia are entirely negligible. To guide the experiment design and analyze the results follow the steps indicated below.



1. Use dimensional analysis to express the lift force F as a function of the minimum number of dimensionless parameters.

$F = F(f, L, \rho, \mu)$

$\frac{F}{\rho L^4 f^2} = \Pi\left(\frac{\rho L^2 f}{\mu}\right)$

2. Assuming the following characteristic values for the fly
   - Flapping frequency: $f = 200$ Hz
   - Wing length: $L = 2.5$ mm
   - Air density: $\rho = 1.2$ kg/m^3
   - Air viscosity: $\mu = 1.8\times10^{-5}$ kg/m·s

   along with the nominal characteristics of the experiment
   - Flapping frequency: $f_e = 0.2$ Hz
   - Wing length: $L_e = 250$ mm
   - Mineral oil density: $\rho_e = 800$ kg/m^3,

   determine the mineral-oil viscosity μ_e needed to guarantee that the experiment is a physically similar representation of the fly dynamics.

$\frac{\rho}{\mu} L^2 f = \frac{\rho_e}{\mu_e} L_e^2 f_e$

$\mu_e = \mu \frac{\rho_e}{\rho}\left(\frac{L_e}{L}\right)^2 \frac{f_e}{f} =$

$= 1.8\cdot10^{-5} \frac{kg}{m\,s} \frac{800}{1.2}\left(\frac{250}{2.5}\right)^2 \frac{0.2}{200} = 0.12 \frac{kg}{m\,s}$

3. The experimental facility including the model with $L_e = 250$ mm and the mineral oil selected above is used to investigate the variation of the lift with the flapping frequency. To that end, the wings are moved at different frequencies f_e and the corresponding forces F_e are measured, yielding the data points shown in the table below. Generate a curve for the variation of the force F with the flapping frequency f for the fly of wing length L =2.5 mm.

f_e (Hz)	F_e (N)
0.14	1.33
0.15	3.33
0.16	4.80
0.17	7.13
0.18	8.00
0.19	6.60
0.20	5.07
0.21	4.53

$\frac{\rho L^2 f}{\mu} = \frac{\rho_e L_e^2 f_e}{\mu_e} \Rightarrow f = \frac{\rho_e}{\rho}\left(\frac{L_e}{L}\right)^2 \frac{\mu}{\mu_e} f_e$

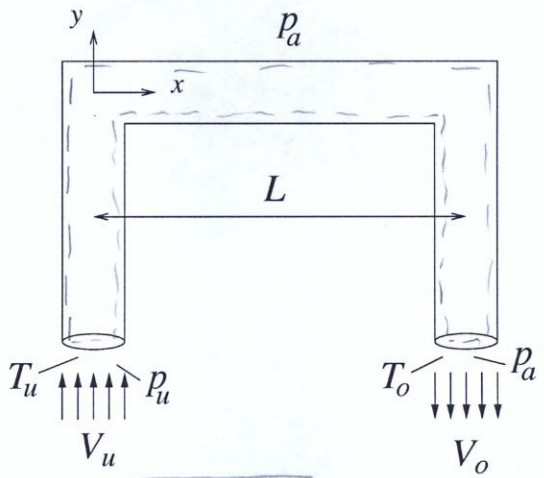
$\frac{F}{\rho L^4 f^2} = \frac{F_e}{\rho_e L_e^4 f_e^2} \Rightarrow F = \frac{\rho}{\rho_e}\left(\frac{L}{L_e}\right)^4 \left(\frac{f}{f_e}\right)^2 F_e = \frac{\rho_e}{\rho}\left(\frac{\mu}{\mu_e}\right)^2 F_e$

f (Hz)	F (N)
140	0.2×10^{-4}
150	0.5×10^{-4}
160	0.72×10^{-4}
170	1.07×10^{-4}
180	1.2×10^{-4}
190	1×10^{-4}
200	0.76×10^{-4}
210	0.68×10^{-4}

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The gas circulating along the pipe of cross section A depicted in the figure discharges to the atmosphere, where the pressure is p_a. Measurements at a given upstream location give V_u, p_u, and T_u for the velocity, pressure and temperature, all uniform across the section. In the analysis that follows, it is assumed that the pipe is thermally insulated and that buoyancy forces have a negligible effect on the gas motion. The gas properties are defined by its gas constant R_g together with the ratio of specific heats $\gamma = c_p/c_v$.

- Obtain the values of the gas temperature and velocity at the outlet section T_o and V_o in terms of V_u, p_u, T_u, p_a, R_g and γ).
- Compute the two components of the force F_x and F_y acting on the stretch of pipe considered.
- Calculate the momentum on the pipe with respect to the origin of the coordinate system indicated in the figure.

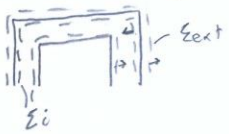


$\rho_u = \frac{p_u}{R_g T_u}$

CONTINUITY $\rho_u V_u A = \rho_o V_o A \Rightarrow \boxed{\frac{p_u V_u}{T_u} = \frac{p_a V_o}{T_o}}$

ENERGY $\int_{\Sigma_c} \rho\left(e+\frac{V^2}{2}\right)\bar{v}\cdot\bar{n}\,d\sigma = -\int_{\Sigma_c} p\,\bar{v}\cdot\bar{n}\,d\sigma + \int_{\Sigma_c} \bar{v}\cdot\bar{\bar{\tau}}\cdot\bar{n}\,d\sigma \Rightarrow \boxed{\frac{\gamma}{\gamma-1}R_g T_u + \frac{1}{2}V_u^2 = \frac{\gamma}{\gamma-1}R_g T_o + \frac{1}{2}V_o^2}$

$\bar{F} = -\int_{\Sigma_{int}} (p-p_a)\,\bar{n}\,d\sigma + \int_{\Sigma_{int}} \bar{\bar{\tau}}'\cdot\bar{n}\,d\sigma$



MOMENTUM CONSERVATION IN THE CONTROL VOLUME

$\int_{\Sigma_c} \rho\,\bar{v}\,\bar{v}\cdot\bar{n}\,d\sigma = -\int_{\Sigma_c} (p-p_a)\,\bar{n}\,d\sigma + \int_{\Sigma_c} \bar{\bar{\tau}}'\cdot\bar{n}\,d\sigma$

$-\rho_u V_u A\,V_u\,\bar{e}_y - \rho_u V_u A\,V_o\,\bar{e}_y = -\int_{\Sigma_w} (p-p_a)\,\bar{n}\,d\sigma + (p_u-p_a)\bar{e}_y A + \int_{\Sigma_w} \bar{\bar{\tau}}'\cdot\bar{n}\,d\sigma$

$\boxed{\bar{F} = \left[\rho_u V_u A\,(V_u+V_o) + (p_u-p_a)A\right]\bar{e}_y}$

$\bar{M} = -\int_{\Sigma_{int}} (p-p_a)\,\bar{x}\wedge\bar{n}\,d\sigma + \int_{\Sigma_{int}} \bar{x}\wedge\bar{\bar{\tau}}'\cdot\bar{n}\,d\sigma$

CONSERVATION OF ANGULAR MOMENTUM IN THE CONTROL VOLUME

$\int_{\Sigma_c} (\bar{x}\wedge\rho\bar{v})\,\bar{v}\cdot\bar{n}\,d\sigma = -\int_{\Sigma_c} \bar{x}\wedge p\bar{n}\,d\sigma + \int_{\Sigma_c} \bar{x}\wedge\bar{\bar{\tau}}'\cdot\bar{n}\,d\sigma$

$-\rho_u V_u A L V_o\,\bar{e}_z = -\int_{\Sigma_w} \bar{x}\wedge(p-p_a)\,\bar{n}\,d\sigma + \int_{\Sigma_w} \bar{x}\wedge\bar{\bar{\tau}}'\cdot\bar{n}\,d\sigma$

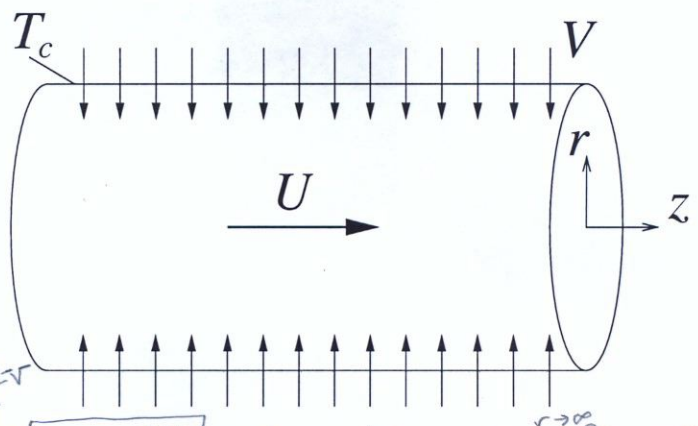
$\boxed{\bar{M} = \rho_u V_u A L V_o\,\bar{e}_z}$

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An infinitely long porous cylinder of radius R is surrounded by a liquid of density ρ, viscosity μ, specific heat c, and thermal conductivity k. The cylinder moves axially with velocity $v_z = U$ and entrains the liquid through its surface pores with a suction velocity $v_r = -V$, inducing (for $r>R$) an axisymmetric velocity field with components $v_r(r)$, $v_z(r)$ and $v_\theta = 0$. At large radial distances $r \gg R$ from the cylinder the liquid is at rest with uniform pressure p_∞. Assume in the following analysis that the effect of gravity on the fluid motion can be neglected.

1. Use the continuity equation to determine the radial velocity component $v_r(r)$ for $R \leq r < \infty$.
2. From the radial component of the momentum equation compute the pressure distribution, demonstrating in particular that $\partial p/\partial z = 0$.
3. Determine the axial velocity distribution $v_z(r)$ as a function of the suction Reynolds number $Re = \rho V R/\mu$ (to solve the problem, assume that the two independent solutions of the resulting linear ODE are of the form $v_z \propto r^\alpha$, obtaining the two values of α that are compatible with the equation).
4. Obtain the force acting on the cylinder (per unit length).
5. Calculate the trajectory of the fluid particle located initially at $r = r_o$ and $z = 0$.
6. Compute the temperature field $T(r)$ when the cylinder temperature T_c is different from that of the surrounding atmosphere $T_\infty > T_c$. Neglect viscous dissipation and express the result with use made of the Prandtl number $Pr = \mu c/k$.
7. Determine the heat transfered to the cylinder per unit time (per unit length).

p_∞ T_∞



1) $\nabla\cdot\bar{v}=0 \Rightarrow \frac{\partial(rv_r)}{\partial r}=0 \Rightarrow \boxed{v_r = -\frac{RV}{r}}$ 2) $\rho v_r\frac{dv_r}{dr} = -\frac{dP}{dr} \overset{r\to\infty,\ P\to P_\infty}{\Rightarrow} \boxed{P = P_\infty - \frac{\rho R^2V^2}{2r^2}}$ $\frac{\partial P}{\partial z}=0$

3) z-Mom: $\rho v_r\frac{dv_z}{dr} = \mu\frac{1}{r}\frac{d}{dr}\left(r\frac{dv_z}{dr}\right) \Rightarrow \frac{d}{dr}\left(r\frac{dv_z}{dr} + Re\, v_z\right)=0$ $r\frac{dv_z}{dr} = -Re\, v_z$

4) $\boxed{\bar{F} = \int_{\Sigma_c}(\bar{\bar{\tau}}'\cdot\bar{e}_r)\,d\sigma = -\mu U\frac{\rho VR}{\mu R}\,2\pi R\cdot 1\,\bar{e}_z = -\rho UV\,2\pi R\,\bar{e}_z}$ $\boxed{v_z = C r^{-Re} = U\left(\frac{r}{R}\right)^{-Re}}$

$\hookrightarrow \bar{\bar{\tau}}'\cdot\bar{e}_r\big|_{r=R} = (\tau'_{rr}, \tau'_{r\theta}, \tau'_{rz})_{r=R} = \left(2\mu\frac{dv_r}{dr}\Big|_{r=R}, 0, \mu\frac{dv_z}{dr}\Big|_{r=R}\right)$

5) $\frac{dr}{dt} = v_r = -\frac{RV}{r} \rightarrow \boxed{r^2 - r_o^2 = -2RVt}$ $\frac{dz}{dt} = v_z$ $\frac{dr}{dz} = \frac{v_r}{v_z} \Rightarrow \boxed{\frac{z}{R} = \frac{U}{(2-Re)V}\left[\left(\frac{r_o}{R}\right)^{2-Re} - \left(\frac{r}{R}\right)^{2-Re}\right]}$

$\hookrightarrow -\mu U Re$

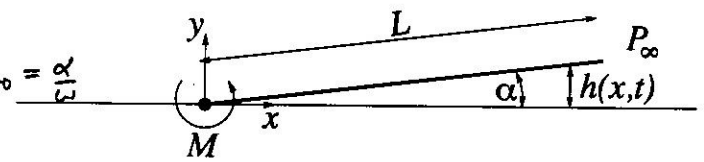
6) $\rho c v_r\frac{dT}{dr} = \cancel{\Phi_v} + \frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)$ $\begin{cases} T(R) = T_c \\ T(\infty) = T_\infty \end{cases}$ $\frac{T-T_\infty}{T_c-T_\infty} = \left(\frac{r}{R}\right)^{-Pr\,Re}$

7) $\boxed{\left(k\frac{dT}{dr}\right)_{r=R} 2\pi R} = \boxed{2\pi\rho c V R\,(T_\infty - T_c)}$

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VF 13.7 A plate of length L is initially sitting on a horizontal plane in the presence of a stagnant liquid atmosphere with reduced pressure P_∞. At a given instant, we begin to rotate the plate with constant angular velocity $\omega = d\alpha/dt$ by applying a given torque M at its left end, as sketched in the figure. For the analysis, use the approximation $h(x,t) = x\tan(\alpha) \simeq x\alpha$, valid for $\alpha \ll 1$.

1. Demonstrate that for values of α sufficiently smaller than a critical value, to be determined, the fluid motion in the gap formed between the plate and the wall is dominated by viscosity.
2. Obtain the velocity profile v_x in the gap as a function of the unknwon value of $P_l(x,t) = -\partial P/\partial x$ as well as the associated volume flux at a given section $Q = \int_0^{\alpha x} v_x dy$.
3. Using continuity, write an equation linking Q and ω, and integrate it to compute the pressure distribution $P(x,t)$ (in the integration, you may anticipate that $x^3 P_l \to 0$ as $x \to 0$).
4. Determine the torque $M(t)$ needed to provide a constant angular velocity ω.



$h_c \sim L\alpha$

$\omega = \frac{d\alpha}{dt} \Rightarrow t_0 = \frac{\alpha}{\omega}$

1) $\frac{h_c^2/(\mu l_3)}{t_0} \sim \frac{L^2\alpha^2\omega}{\nu\alpha} \ll 1 \Rightarrow \boxed{\alpha \ll \frac{\nu}{L^2\omega}}$

$\frac{V_L h_c}{\nu}\frac{h_c}{L} \sim \frac{\omega L^2}{\nu}\frac{h_c}{L} \sim \frac{\omega L^2\alpha}{\nu} \ll 1$ ⇑

$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \to V_L \sim \frac{\omega L^2}{h_c}$

2) $V_x = \frac{P_l}{2\mu} y(h-y) \to Q = \frac{P_l}{12\mu} h^3 = -\frac{x^3\alpha^3}{12\mu}\frac{\partial P}{\partial x}$

3) $\int_0^h \frac{\partial V_x}{\partial x} dy + \int_0^h \frac{\partial}{\partial y} V_y\, dy = 0 \Rightarrow \frac{\partial}{\partial x} Q - \frac{dh}{dx} V_x(y=h) + V_y(h) - V_y(0) = 0$

$\frac{\partial Q}{\partial x} + \omega x = 0$

$\frac{\partial}{\partial x}\left(\frac{\alpha^3}{12\mu} x^3 \frac{\partial P}{\partial x}\right) - \omega x = 0 \Rightarrow \frac{\alpha^3}{12\mu} x^3 \frac{\partial P}{\partial x} - \omega\frac{x^2}{2} = 0$

$\frac{\partial P}{\partial x} = \frac{6\mu\omega}{\alpha^3}\frac{1}{x} \Rightarrow \boxed{P - P_\infty = \frac{6\mu\omega}{\alpha^3}\ln\left(\frac{x}{L}\right)}$

4) $\bar{M}_{F\to Plate} = -\int_{\Sigma_P} (P-P_\infty)\,\bar{x}\wedge\bar{n}\, d\sigma = \int_0^L \frac{6\mu\omega}{\alpha^3} x \ln\frac{x}{L}\, dx\, \bar{e}_z = -\frac{3}{2}\frac{\mu\omega L^2}{\alpha^3}\bar{e}_z$

TO COUNTERBALANCE THIS MOMENTUM WE NEED TO APPLY A (POSITIVE) TORQUE $\boxed{\bar{M} = \frac{3}{2}\frac{\mu\omega L^2}{\alpha^3}\bar{e}_z}$