INTRODUCTION TO FLUID MECHANICS — Jan 10, 2012

The power W needed to move the propeller of a small airplane depends on the airplane velocity V, propeller diameter D, angular velocity Ω, and also on the air properties, including its density ρ, viscosity μ, and sound velocity c; the latter, characterizing compressibility effects, has dimensions $[c] = LT^{-1}$. Assuming that the effect of gravity is negligible:

1. Use dimensional analysis to express the propeller power W as a function of the minimum number of dimensionless parameters.

A new airplane with propeller of diameter $D = 1.5$ m is to be designed to fly at an altitude $z = 5000$ m with cruise velocity $V = 300$ Km/h and propeller angular velocity $\Omega = 2500$ rpm. To determine the needed power, an experimental model is to be tested in a wind tunnel at sea level ($z = 0$ m).

2. Determine the model diameter D_e, the propeller angular velocity Ω_e, and the wind-tunnel velocity V_e needed to guarantee that the experiment provides a physically similar representation of the airplane flight conditions.

3. For the values of D_e, Ω_e, and V_e determined above, the power measured in the experiment (obtained as the product of Ω_e and the torque acting on the propeller shaft) is $W_e = 6 \times 10^7$ Watt. Obtain the power W corresponding to the real airplane.

Note: Physical properties for air according to the standard atmosphere:

z	T	ρ	μ	c
m	K	kg/m^3	kg/(m s)	m/s
0	288.15	1.225	1.775×10^{-5}	340.3
5000	255.65	0.736	1.630×10^{-5}	320.5

1) $W = W(\rho, \mu, c, V, D, \Omega) \Rightarrow \frac{W}{\rho V^3 D^2} = f\left(\frac{\rho V D}{\mu}, \frac{V}{c}, \frac{\Omega D}{V}\right)$

2) $\frac{\rho V D}{\mu} = \frac{\rho_e V_e D_e}{\mu_e} \Rightarrow D_e = \frac{\mu_e}{\mu}\frac{\rho}{\rho_e}\frac{V}{V_e} D = \frac{\mu_e}{\mu}\frac{\rho}{\rho_e}\frac{c}{c_e} D = 0.924$ m

$\frac{V}{c} = \frac{V_e}{c_e} \Rightarrow V_e = \frac{c_e}{c} V = 319$ Km/h

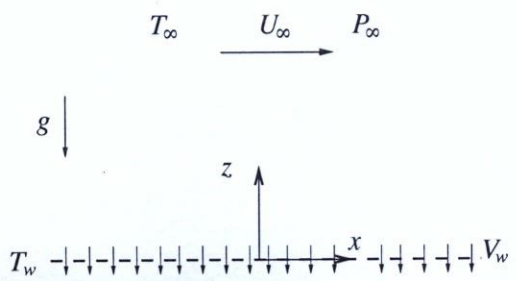
$\frac{\Omega D}{V} = \frac{\Omega_e D_e}{V_e} \Rightarrow \Omega_e = \frac{D}{D_e}\frac{V_e}{V}\Omega = 4308$ rpm

3) $\frac{W}{\rho V^3 D^2} = \frac{W_e}{\rho_e V_e^3 D_e^2} \Rightarrow W = \frac{\rho}{\rho_e}\left(\frac{V}{V_e}\right)^3\left(\frac{D}{D_e}\right)^2 W_e = 7.9 \times 10^7$ Watt

NS1

Consider the steady planar motion of a perfect liquid of density ρ, viscosity μ, thermal conductivity k and specific heat c in the vicinity of a horizontal porous wall. Far from the wall, the horizontal velocity component is $v_x = U_\infty$, the pressure is given by $p + \rho g z = P_\infty$ and the temperature is $T = T_\infty$, different from the value T_w at the wall. The liquid is sucked through the pores of the wall, resulting in a transverse velocity $v_z = -V_w$ at the wall surface. In the analysis, assume that the velocity and temperature fields are only a function of the distance to the wall z.

1. Determine the velocity components $v_x(z)$ and $v_z(z)$ and well as the pressure field $p(x, z)$.
2. Obtain the temperature field $T(z)$
3. Calculate the streamline that crosses $x = x_o$ and $z = 0$.
4. Compute the viscous stress on the wall.
5. Determine the heat flux from the fluid to the wall.



1) $\frac{\partial}{\partial x} = 0$, $V_y = 0$

$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \Rightarrow V_z = \text{CONSTANT} = -V_w$

x-MOM: $\rho V_z \frac{\partial V_x}{\partial z} = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 V_x}{\partial z^2} \Rightarrow \frac{d^2 V_x}{dz^2} + \frac{V_w}{\nu}\frac{dV_x}{dz} = 0 \rightarrow V_x = C_1 + C_2 e^{-\frac{V_w}{\nu} z} = U_\infty \left(1 - e^{-\frac{V_w}{\nu} z}\right)$

z-MOM: $0 = -\frac{\partial P}{\partial z} - \rho g \Rightarrow P + \rho g z = f(x) \overset{z \to \infty}{=} P_\infty$

2) $-\rho c V_w \frac{dT}{dz} = \phi_r + k \frac{d^2 T}{dz^2} \rightarrow T = B_1 + B_2 e^{-\frac{V_w}{\alpha} z} + T_P$

$\phi_r = \mu \left(\frac{dV_x}{dz}\right)^2 = \mu U_\infty^2 \frac{V_w^2}{\nu^2} e^{-2\frac{V_w}{\nu} z}$

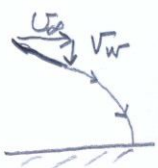
$T_P \rightarrow \frac{Pr\, U_\infty^2 / c}{2(Pr - 2)} e^{-2\frac{V_w}{\nu} z}$

$T(\infty) = T_\infty \rightarrow B_1 = T_\infty$

$T(0) = T_w \rightarrow B_2 = T_w - T_\infty - \frac{Pr}{2(Pr-2)} \frac{U_\infty^2}{c}$

$$T = T_\infty + \left(T_w - T_\infty - \frac{Pr}{2(Pr-2)} \frac{U_\infty^2}{c}\right) e^{-\frac{V_w}{\alpha} z} + \frac{Pr}{2(Pr-2)} \frac{U_\infty^2}{c} e^{-2\frac{V_w}{\nu} z}$$

3) $\frac{dx}{V_x} = \frac{dz}{V_z} \Rightarrow -V_w \int_{x_o}^{x} dx = \int_0^z U_\infty \left(1 - e^{-\frac{V_w}{\nu} z}\right) dz \Rightarrow x - x_o = -\frac{U_\infty}{V_w}\left[z + \frac{\nu}{V_w}\left(e^{-\frac{V_w}{\nu} z} - 1\right)\right]$



4) $\bar{n}$ $\quad \bar{\bar{\tau}}' \cdot \bar{n} = \begin{bmatrix} \tau'_{xx} & \tau'_{xz} \\ \tau'_{xz} & \tau'_{zz} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \left(\tau'_{xz}, \tau'_{zz}\right)_{z=0}$

$\tau'_{xz} = \mu \frac{dV_x}{dz} = \mu \frac{V_w U_\infty}{\nu} e^{-\frac{V_w}{\nu} z}\Big|_{z=0} = \rho V_w U_\infty$

$\tau'_{zz} = 0$

$$\left(\bar{\bar{\tau}}' \cdot \bar{n}\right)_w = \rho V_w U_\infty \bar{e}_x$$

5) $\bar{n}$ $\quad \left(\bar{q} \cdot \bar{n}\right)_w = -k \left(\nabla T \cdot \bar{n}\right)_w = k \frac{dT}{dz}\Big|_{z=0} \Rightarrow \left(\bar{q} \cdot \bar{n}\right)_w = \rho c V_w \left[T_\infty - T_w + \frac{1}{2} \frac{U_\infty^2}{c}\right]$

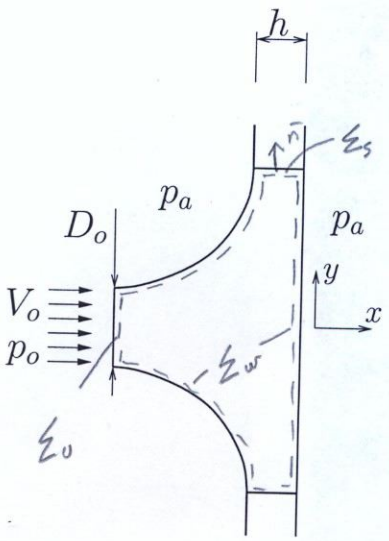
INTRODUCTION TO FLUID MECHANICS

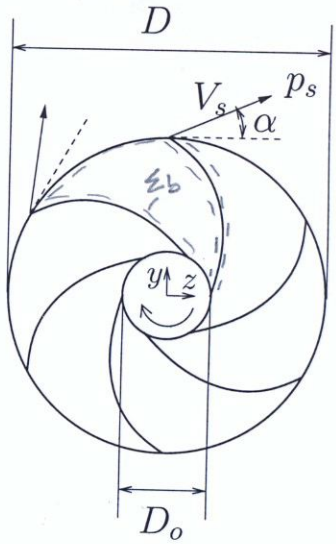
Jan 10, 2012

The figure shows a sketch of a centrifugal pump. Water enters the pump with uniform axial velocity V_o and pressure p_o through a circular pipe of diameter D_o. The internal geometry of the pump includes rotating blades that redirect the flow, so that the water leaves the pump with zero axial velocity through a lateral exit of width h and diameter D. At the exit section, the pressure is $p_s \neq p_a$ and the velocity vector, of magnitude V_s, forms an angle α with the lateral surface, as indicated in the figure, i.e., the corresponding normal and tangential velocity components are $V_s \sin\alpha$ and $V_s \cos\alpha$, respectively. Assuming that the effect of gravity is negligible and that the water temperature at the exit is equal to that found at the entrance determine the values of the

→ THAT THE PUMP WALLS ARE INSULATED

- Radial velocity component at the exit section $V_s \sin\alpha$.
- Axial force exerted on the pump.
- Torque exerted on the pump.
- Power of the pump, assumed to be ideal (i.e., work per unit time exerted on the water stream by the rotating parts).

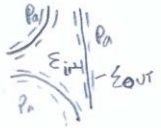




Note: besides the water density ρ and the ambient pressure p_a, the following quantities can be assumed to be known in solving the problem: D, D_o, h, α, V_o, p_o, and p_s.

CONTINUITY: $\int_{\Sigma_c} \rho(\vec{v}-\vec{v}_c)\cdot\vec{n}\,d\sigma = 0 \Rightarrow -\rho V_o \pi \frac{D_o^2}{4} + \rho V_s \sin\alpha\, \pi D h = 0 \Rightarrow \boxed{V_s \sin\alpha = \frac{D_o^2}{4Dh} V_o}$

FORCE



$\vec{F} = -\int_{\Sigma_{IN}+\Sigma_b} p\vec{n}\,d\sigma + \int_{\Sigma_{IN}+\Sigma_b} \vec{\tau}'\cdot\vec{n}\,d\sigma - \int_{\Sigma_{OUT}} p_a\vec{n}\,d\sigma = -\int_{\Sigma_{IN}+\Sigma_b} (p-p_a)\vec{n}\,d\sigma + \int_{\Sigma_{IN}+\Sigma_b} \vec{\tau}'\cdot\vec{n}\,d\sigma$

MOMENTUM

$\int_{\Sigma_c} \rho\vec{v}(\vec{v}-\vec{v}_c)\cdot\vec{n}\,d\sigma = -\int_{\Sigma_c} p\vec{n}\,d\sigma + \int_{\Sigma_c} \vec{\tau}'\cdot\vec{n}\,d\sigma$ ($-\int (p-p_a)\vec{n}\,d\sigma$)

$\Sigma_o + \Sigma_s + \Sigma_w + \Sigma_b$ $\quad$ $\Sigma_o + \Sigma_s + \Sigma_w + \Sigma_b$

× COMPONENT

$\Rightarrow -\rho V_o \pi \frac{D_o^2}{4} V_o \vec{e}_x - (p_o - p_a)\pi \frac{D_o^2}{4}\vec{e}_x = -\int_{\Sigma_w+\Sigma_b} p\vec{n}\,d\sigma + \int_{\Sigma_w+\Sigma_b} \vec{\tau}'\cdot\vec{n}\,d\sigma = -\vec{F}$

$\boxed{\vec{F} = \frac{\pi D_o^2}{4}\vec{e}_x\left[\rho V_o^2 + (p_o - p_a)\right]}$

TORQUE: APPLYING ANGULAR MOMENTUM WITH RESPECT TO THE ORIGIN

$\int_{\Sigma_c} (\vec{x}-\vec{x}_o)\wedge\rho\vec{v}\,(\vec{v}-\vec{v}_c)\cdot\vec{n}\,d\sigma = -\int_{\Sigma_c} (\vec{x}-\vec{x}_o)\wedge(p-p_a)\vec{n}\,d\sigma + \int_{\Sigma_c} (\vec{x}-\vec{x}_o)\wedge\vec{\tau}'\cdot\vec{n}\,d\sigma$

$\boxed{\rho V_o \pi \frac{D_o^2}{4} V_s \cos\alpha \frac{D}{2}\vec{e}_x = -\vec{M}}$

ENERGY EQUATION

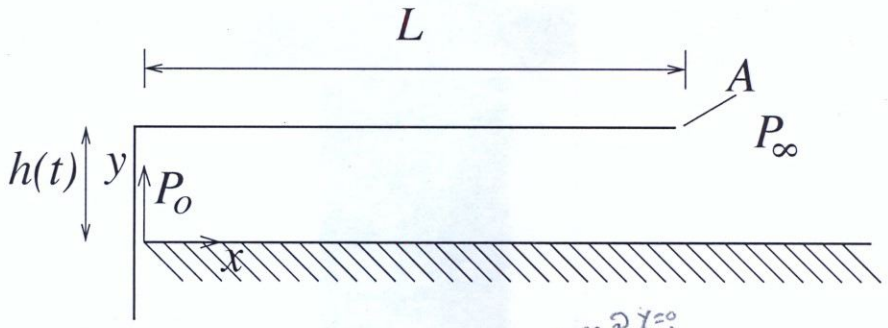
$\int_{\Sigma_c} \rho\left(e + \frac{v^2}{2}\right)(\vec{v}-\vec{v}_c)\cdot\vec{n}\,d\sigma = -\int_{\Sigma_c} p\vec{v}\cdot\vec{n}\,d\sigma + \int_{\Sigma_c} \vec{v}\cdot\vec{\tau}'\cdot\vec{n}\,d\sigma + \int_{\Sigma_c} k\nabla T\cdot\vec{n}\,d\sigma$

$\rho V_o \pi \frac{D_o^2}{4}\left(\frac{V_s^2}{2} - \frac{V_o^2}{2}\right) = +V_o \pi \frac{D_o^2}{4}(p_o - p_s) + W \Rightarrow \boxed{W = \rho V_o \pi \frac{D_o^2}{4}\left[\left(\frac{p_s}{\rho} + \frac{V_s^2}{2}\right) - \left(\frac{p_o}{\rho} + \frac{V_o^2}{2}\right)\right]}$

INTRODUCTION TO FLUID MECHANICS — Jan 10, 2012

The L-shaped arm of length L depicted in the figure moves vertically approaching the horizontal wall, forming a thin channel of decreasing uniform thickness $h(t) \ll L$. A liquid of density ρ and viscosity μ fills the channel. Due to the arm displacement, the liquid is forced to move out, discharging to the surrounding atmosphere, where the reduced pressure is P_∞. For a known value of $h(t)$,

1. Give the conditions for the motion in the channel to be dominated by viscosity.
2. Determine the velocity profile across the channel $v_x(x, y, t)$, the associated volume flux $Q = \int_0^h v_x dy$, and the reduced pressure distribution $P(x, t)$, including the value $P_o(t)$ at $x = 0$.
3. Calculate the force acting on the arm $\bar{F} = (F_x, F_y)$.
4. Obtain the torque exerted on the arm with respect to its end point A.



1) $\frac{\rho V_L h}{\mu}\frac{h}{L} \ll 1$, $V_L \sim \frac{L}{h}\dot{h} \Rightarrow \boxed{\frac{h\dot{h}}{\nu} \ll 1}$

$\frac{h^2/\nu}{t_o} \ll 1 \Rightarrow \boxed{\frac{h\dot{h}}{\nu} \ll 1}$

$\hookrightarrow t_o \simeq \frac{h}{\dot{h}}$

2) $0 = P_\rho + \mu \frac{\partial^2 V_x}{\partial y^2} \Rightarrow \boxed{V_x = \frac{P_\rho}{2\mu} y(h-y)}$ ($V_x = 0$ @ $y = 0$, $y = h$)

$Q = P_\rho \frac{h^3}{12\mu} = -\frac{\partial P}{\partial x}\frac{h^3}{12\mu}$

$\int_0^{h(t)} \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0\right) dy \Rightarrow \frac{\partial Q}{\partial x} + \frac{dh}{dt} = 0 \Rightarrow \boxed{-\frac{\partial}{\partial x}\left(P_\rho \frac{h^3}{12\mu}\right) + \frac{dh}{dt} = 0}$

$-Q = \frac{\partial P}{\partial x}\frac{h^3}{12\mu} = \frac{dh}{dt}x + C_1 \Rightarrow \boxed{P = \frac{12\mu}{h^3}\frac{dh}{dt}\frac{x^2}{2} + C_2 = \frac{6\mu}{h^3}\frac{dh}{dt}(x^2 - L^2) + P_\infty}$

($C_1 = 0$, BECAUSE $Q = 0$ AT $x = 0$; $x = L$, $P = P_\infty$)

$x = 0$, $P = \boxed{P_o = -\frac{6\mu}{h^3}\frac{dh}{dt}L^2 + P_\infty}$

3) $\boxed{F_y = \int_0^L (P - P_\infty)\, dx = -\frac{4\mu}{h^3}\frac{dh}{dt}L^3}$

$\boxed{F_x = -(P_o - P_\infty)h - \int_0^L \mu \left.\frac{\partial V_x}{\partial y}\right|_{y=h} dx = \frac{3\mu}{h^2}\frac{dh}{dt}L^2}$

$\frac{6\mu}{h^2}\frac{dh}{dt}L^2$; $-\frac{3\mu}{h^2}\frac{dh}{dt}L^2$

4) $\boxed{\bar{M} = -\int_0^L (P - P_\infty)(L - x)\, dx\, \bar{e}_z = -\int_0^L \frac{6\mu}{h^3}\frac{dh}{dt}(x^2 - L^2)(L - x)\, dx\, \bar{e}_z = +\frac{5}{2}\frac{\mu}{h^3}\frac{dh}{dt}L^4 \bar{e}_z}$